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# RAMAN SCATTERING IN HIGH-T<sub>c</sub> SUPERCONDUCTORS

### Manuel CARDONA

Max-Planck-Institut für Festkörperforschung, Heisenbergstr. 1, 7000 Stuttgart 80, Federal Republic of Germany

The use of Raman Spectroscopy for the investigation and characterization of high  $T_c$  superconductors is reviewed. A parallel is drawn with some of the effects observed for conventional heavily doped semiconductors. Recent advances are discussed.

### 1. INTRODUCTION

Raman spectroscopy gained a prominent role in the investigation of high  $T_c$  superconductors (high  $T_c$  SC's) shortly after their discovery.<sup>1,2</sup> The initial work focussed on phonon eigenvectors,<sup>3</sup> and frequencies and their use for characterizing isotopic substitution.<sup>1</sup> Investigation of phonon anomalies at  $T_c$  followed.<sup>4,5</sup> The symmetry properties of  $B_{1g}$  phonons of Y-123 at ~340 cm<sup>-1</sup> were used to identify single domain single crystals.<sup>5</sup> An electronic scattering background, in which a gap opens below  $T_c$ , was also early discovered.<sup>5-9</sup> This background was shown to interfere coherently with the phonons, giving rise to Fano-type asymmetric Raman lineshapes.<sup>5</sup> Scattering by two magnons was also early identified in the insulating modifications of high  $T_c$  SC's.<sup>10,11</sup>

Recently, absolute scattering efficiencies  $I_S$  of Y-123 have been measured and compared favorably with results of LMTO LDA "ab initio" calculations of the Raman tensor.<sup>12</sup> These results have yielded quantitative information on electronphonon coupling constants which can also be obtained from the changes in the real<sup>13</sup> and imaginary<sup>14</sup> parts of phonon self-energies (SE) at  $T_c$ . For the Raman (and also ir) phonons the electron-phonon coupling constants  $\lambda$  are, at most, onehalf of those required to explain  $T_c$ 's higher than 77 K. These measurements also yield plausible values of the gap  $(2\Delta \simeq 5kT_c)$  and suggest the presence of more than one gap or a distribution of gaps around the Fermi surface (Y-124, Ref. 15). This can also be inferred from the electronic scattering background.<sup>15</sup> Raman scattering also shows rather normal  $\lambda$ 's for the apical oxygen vibrations,<sup>16</sup> a fact which speaks against a dominant role in the high- $T_c$  mechanism. Anomalies in  $I_S$  at  $T_c$  have also been observed in Y-124<sup>15</sup> and Y-12317 and attributed to resonant scattering across the superconducting gap. Recently a controversy has arisen about whether in Y-123 the  $B_{1g}$  phonons (340 cm<sup>-1</sup>) sharpen<sup>18,19</sup> or broaden<sup>14</sup> below  $T_c$ . This controversy has been solved: sharpening below  $T_c$  only happens in samples with strong substitutional doping into the Cu sites.<sup>20</sup>

Scattering by strong coupled phonon-f-electron excitations (of the Nd-ion) has been reported in Nd-123.<sup>21,22</sup> The dependence on temperature of the doublet observed has been related to occupation statistics of the f-electron crystal field levels. Another recent development is the observation of iractive (Raman forbidden) LO modes in the Raman spectra of YBa<sub>2</sub>Cu<sub>3</sub>O<sub>6</sub> under strongly resonant conditions. They are believed to be "quadrupole allowed" by the strong Fröhlich interaction which accompanies LO phonons.

Several reviews of Raman scattering in high- $T_c$  SC's have appeared.<sup>23-28</sup>

## 2. ELECTRONIC SCATTERING

Raman scattering by free carriers has been extensively studied in heavily doped semiconductors.<sup>29, 32</sup> Free electrons couple to the scattering Hamiltonian through the  $\vec{A} \cdot \vec{p}$  term  $(\vec{A} = \vec{A}_L + \vec{A}_S)$ , the sum of the incident and scattered vector potentials) in second order perturbation theory and the  $A^2$ term in first order. Away from resonances, and using the  $\vec{k} \cdot \vec{p}$ expression for the free carrier effective mass, the scattering Hamiltonian becomes<sup>29</sup>

$$\vec{A}_L \cdot \frac{1}{\vec{m}} \cdot \vec{A}_S \tag{1}$$

where  $\vec{m}$  is the mass tensor of the carriers. If all these carriers have the same  $\vec{m}$  Eq. (1) behaves as a longitudinal perturbation inducing a charge density wave which is screened by the carriers. This result in the Stokes efficiency for a carrier concentration N per unit solid angle and unit path length:<sup>32</sup>

$$I_{S}(\Omega,q) = -\left(\frac{\omega_{S}}{\omega_{L}}\right)^{2} \tau_{0}^{2} \left(\hat{\epsilon}_{L} \cdot \frac{m_{0}}{\bar{m}} \cdot \hat{\epsilon}_{S}\right)^{2} + (1+n(\Omega)) \cdot \frac{\hbar q^{2}}{4\pi\epsilon^{2}} \cdot Im \frac{1}{\epsilon(\Omega)} d\Omega$$

$$(2)$$

where  $r_0 = \epsilon^2/m_0c^2$  (Thomson radius),  $\Omega = \omega_L - \omega_S$  is the Stokes shift  $n(\Omega)$  the corresponding Bose-Einstein factor and  $\varepsilon(\Omega)$  the dielectric function, including free and bound carrier



FIGURE 1 Raman scattering by intervalley density fluctuations in *n*-type silicon fitted with Eq. (3).<sup>30</sup>



FIGURE 2 Electronic Raman scattering of high- $T_c$  superconductors fitted with Eqs. (3-6).<sup>8,35</sup>

contributions. Equation (2) yields a peak at the screened free carrier plasma frequency  $\Omega_p$  and a vanishing scattering efficiency for  $\Omega \rightarrow 0, \sim 10^{-5}$  times smaller than the observed one at  $\sim 100$  cm<sup>-1</sup>.

The smallness of the  $I_s$  of Eq. (2) for  $\omega \to 0$  is due to screening of the low frequency charge density fluctuations by the free carriers which lowers the coupling to Eq. (1). This screening can be prevented if the density fluctuations have zero charge and nevertheless couple to Eq. (1). This happens in multicomponent carrier systems, e.g., if the various components have different mass tensors. A typical case is that of heavily doped Si (six equivalent valleys along (100)) or Ge ( four equivalent valleys along (111).<sup>30,31</sup> In these cases there are *intervalley* density fluctuations with zero charge (charge increase in one valley  $\equiv$  decrease in another) which are not screened. They scatter the light strongly at low frequencies if the various mass tensors referred to the same set of axes are very different. Such is the case of *n*-type Si and Ge: for Si, e.g., the mass tensors are uniaxial  $(m_{\parallel} = 0.9 m_0, m_{\perp} = 0.2 m_0)$  and oriented orthogonally to each other for the various valleys. Thus the contributions of these valleys to Eq. (1) do not cancel even if the charge fluctuations do. Scattering by unscreened low frequency density fluctuations results, as shown in Fig. 1.<sup>30</sup> The corresponding Stokes spectra are well represented by (Fig. 1):<sup>30-33</sup>

$$I_{S} = (1 + n(\Omega)) \frac{B\Omega\tau^{-1}}{\Omega^{2} + \tau^{-2}}$$
  
=  $(1 + n(\Omega)) \frac{B\Omega\tau^{-1}}{\Omega^{2}\tau^{2} + 1}$  (3)

where  $\tau^{-1}$  is the intervalley scattering time and:

$$B = \frac{\hbar e^4}{\pi c^4} \frac{dN}{dE_F} \left\langle \left[ \hat{e}_L \cdot \left( \frac{1}{\dot{m}} - \left\langle \frac{1}{\dot{m}} \right\rangle \right) \cdot \hat{e}_S \right]^2 \right\rangle , \quad (4)$$

 $\langle ... \rangle$  being the average over all valleys and  $E_F$  the Fermi energy. The parameter *B* is thus determined by the "fluctuations" of  $\vec{m}$  over the Fermi surface. Note that these fluctuations are suppressed (and the low frequency scattering eliminated) if all carriers are transferred to a single valley by application of uniaxial stress.<sup>31</sup>

In high- $T_c$  SC's low frequency scattering much stronger than that given by Eq. (2) is observed (Fig. 2).<sup>5-8</sup> It is reasonable to attempt an explanation based on neutral carrier density fluctuations related to mass fluctuations around the Fermi surface.<sup>32</sup> There are, however, two features which resist the fit to Eq. (3):

- 1.  $I_S$  is nearly independent of  $\Omega$  over a wide frequency range (it increases by less than a factor of 2 from  $\simeq$ 0.01 to 1 eV). This is contrary to the strong decrease with increasing  $\Omega$  required by Eq. (3).
- 2.  $I_S$  is nearly independent of temperature. This is contrary to the dependence required by the Bose-Einstein factor and observed for *n*-Si and *n*-Ge.<sup>30,31</sup>

One can try to modify Eq. (3) in a phenomenological way so as to reproduce these features.<sup>34</sup> A microscopic derivation of this modification, based on the existence of large nesting parts of the Fermi surface, has been given.<sup>35</sup> It leads to a replacement of  $\tau^{-1}$  in Eq. (3) by the frequency and temperature dependent  $\tau$ :

$$\tau^{-1} = \alpha \cdot Max \left[ |\omega| + \beta' T \right] \tag{5}$$

where  $\alpha \approx 0.5$  and  $\beta' \approx 3.5$  (*T* in units of frequency). This  $\tau^{-1}$  corresponds to an imaginary part of the self-energy (SE) for electron-electron scattering in a "nesting" situation. The



FIGURE 3 Difference in the electronic scattering of YBa<sub>2</sub>Cu<sub>3</sub>O<sub>8</sub> above and below  $T_c$  for two different polarization configurations.<sup>22</sup> These data suggest the presence of two energy gaps.

corresponding real part leads to a renormalization of  $\Omega^2$  in the denominator of Eq. (3) which must be multiplied by a frequency and temperature dependent effective mass:

$$\frac{m^{\bullet}(\omega,T)}{m_0} = 1 + \frac{2\alpha}{\pi} \ell n \frac{\omega_c}{Max(\beta'T,|\Omega|)}$$
(6)

with  $\omega_c$  a cutoff  $\approx 1 \text{ eV}$ . Figure 2 shows a fit with Eqs. (5,6) to data of Staufer et al.<sup>8</sup> using  $\alpha$  as a fitting parameter. The fit above 800 cm<sup>-1</sup> is excellent. It is also good between 200 - 800 cm<sup>-1</sup> if one removes the phonon structures. The divergence of the theory below  $\approx 200 \text{ cm}^{-1}$ , not observed experimentally, may be an artifact of the calculation. It would be of interest to calculate *B* from the mass fluctuation around the Fermi surface for a realistic band structure, to estimate from it with Eqs. (3 - 6) the absolute  $I_S$ , and to compare it with experiments.

The flat electronic continuum of the type of Fig. 2 opens a gap below  $T_c$  which, as shown in Fig. 3 for YBa<sub>2</sub>Cu<sub>4</sub>O<sub>8</sub>, is not sharp. BCS theory for spherical Fermi surfaces requires a sharp gap  $2\Delta \simeq 3.5kT_c$  with zero scattered intensity below  $2\Delta$ . It is hard to decide experimentally whether inelastic scattering is present all the way down to  $\Omega = 0$  or a sharp cutoff takes place. Fano interference of the vibrations which correspond to the 115 cm<sup>-1</sup> phonon of YBa<sub>2</sub>Cu<sub>3</sub>O<sub>7</sub>



FIGURE 4 Raman spectra of intrinsic and heavily doped *p*-type Ge showing a broadening and softening due to electron-phonon interaction.<sup>38</sup>

below  $T_c$  has been reported in Ref. 36. It indicates that at this frequency some coherent continuum  $I_S$  is still available, in agreement with Fig. 3. It is hard to decide, however, whether the universally observed, "soft gap" ("pseudogap") is an intrinsic property of the high  $T_c$  SC's or is rather related to material defects or the presence of the surface. The fact that it is observed by many workers and for different samples may favor the intrinsic nature. In any case, Fig. 3 gives different values  $2\Delta (2\Delta_1 \text{ and } 2\Delta_2)$  for rr and yy polarizations, averaging roughly to the BCS value of 3.5  $kT_c$ !. This suggests variations of  $2\Delta$  over the Fermi surface with stable points at  $2\Delta_2$  and  $2\Delta_1$ .<sup>37</sup>

## **3. PHONON ANOMALIES**

Effects of the electron-phonon interaction can be directly observed in the Raman spectra of phonons by doping conventional semiconductors. Results obtained for *p*-type Ge<sup>38</sup> are shown in Fig. 4. The width of the phonon of the intrinsic material, produced by anharmonic decay into two or more phonons, increases upon doping with Ga while its frequency decreases. This is interpreted as due to the SE of the phonons interacting with the holes in the valence band. The real part of this SE is responsible for the phonon softening while its imaginary part produces the broadening. The latter is only non-zero when *real* electron (or hole) transitions can take place at the phonon frequency while the former. a softening, implies that more virtual transitions can take place above the phonon frequency than below.

Similar effects exist in metals but it is not easy to dis-



FIGURE 5 Lincwidth of the Raman c-stretching mode of the  $O_{IV}$  (apical) oxygen of YBa<sub>2</sub>Cu<sub>3</sub><sup>16</sup>O<sub>7</sub> and DyBa<sub>2</sub>Cu<sub>3</sub><sup>18</sup>O<sub>7</sub> showing the self-energy effect for  $T < T_c$ .<sup>16</sup>



FIGURE 6 Linewidths (normalized to the calculated coupling constant  $\lambda$ ) of several 123 SC's versus phonon frequency.<sup>14</sup>

entangle them from elastic backgrounds because of the impossibility of changing the carrier density. In high- $T_c$  SC's changes in the real and imaginary parts of phonon SE's can be observed through the SC transition.<sup>2,3,14-19</sup> The real part of the SE reflects itself in a softening or hardening below  $T_c$ , depending on whether the phonon lies below or above the gap. In the former case a sharpening occurs as *real* electronic excitations available for the decay of the phonon are removed <u>below</u>  $T_c$ . In the latter, the density of such excitations increases at  $\omega = \Omega$  and so do the phonon widths.

Figure 5 shows the *increase* in the linewidth found below  $T_c$  for the apical oxygen phonon (vibrations of  $O_{IV}$  along

c) of two 123-materials. Above  $T_c$  the standard anharmonic broadening, fitted with the statistical factor  $A(1+2n(\Omega/2))$ , is shown. That *increase* indicates that a gap exists at or below those vibrational frequencies, i.e.  $\leq 480$  cm<sup>-1</sup>.

A detailed analysis of the temperature dependence of the line width is shown in Fig. 6 for the phonons which correspond to vibrations along c of the oxygens in the CuO<sub>2</sub> planes of many 123-materials. Two such Raman active phonons exist, a fully symmetric one at ~ 440 cm<sup>-1</sup> and another at ~ 340 cm<sup>-1</sup> with  $B_{1g}$  tetragonal symmetry. The fit of these data to theory<sup>39</sup> yields  $2\Delta/kT_c \simeq 5.0$  and for the electron phonon coupling constant  $\lambda \simeq 0.5$  (provided all phonons couple with the same strength.<sup>14</sup>). The broadenings below  $T_c$  in Fig. 6 have been contested in Refs. 18,19. The reason seems to be that the samples in<sup>18,19</sup> contained substitutional impurities (Au, Th) which strongly affect the anisotropy of the gaps and change broadenings of the 340 cm<sup>-1</sup> mode into sharpenings (Fig. 7).<sup>20</sup>

Not only the SE's exhibit anomalous behavior at  $T_c$  but



FIGURE 7

Linewidth  $2\gamma$  and frequency shifts with T of the  $B_{1g}$  (tetragonal notation) phonons of  $YBa_2Cu_{3-x}Au_xO_7$  for several values of x. With increasing x the effect on  $\gamma$  evolves from a broadening to a sharpening below  $T_c$ .<sup>20</sup>



FIGURE 8

Resonance profiles of several Raman phonons in absolute Raman efficiency units, compared with LMTO calculations (also absolute!). The dashed lines represent calculation for unmixed ionic motion modes while the solid lines represent fits with mode admixture. Note that 435 mode derives is strength from the mode admixture while the 115 (Ba-like) and 150 cm<sup>-1</sup> (Cu-like) modes seem to be unmixed.<sup>12</sup>

also the  $I_S$ 's: a sharp increase is usually found below  $T_c$  (sometimes a decrease, depending on phonon and  $\omega_L$ ).<sup>17,22</sup> This has also been interpreted as due to a resonance of the phonon frequency with the gap.<sup>17,22</sup>

## 4. RESONANT RAMAN PHENOMENA

Is depends strongly on laser frequency (Fig. 8)<sup>21,40</sup> and exhibits structures characteristic of resonances of  $\omega_L$  and/or  $\omega_S$  with strong interband transitions. The resonance profiles of Fig. 8 were calculated from *ab initio* LMTO band structures by evaluating the dielectric function  $\varepsilon(\omega)$  and its derivative with respect to the phonon coordinate. The agreement obtained between calculated and measured  $I_S$  in absolute units is remarkable. To the best of our knowledge this is the first material with a unit cell of more than 2 atoms for which such theoretical work has been performed. The results give a lot of confidence in electron-phonon coupling constants obtained from band structure calculations. From this work information about the phonon eigenvectors can



FIGURE 9 Resonant Raman spectra of YBa<sub>2</sub>Cu<sub>3</sub>O<sub>6</sub> showing Raman forbidden, ir-active LO modes (wavy arrows).<sup>22</sup>

be obtained. It was shown<sup>12</sup> that best agreement with the measured Raman profile is found for nearly pure Ba-modes (solid line in Fig. 8) while the profiles of the  $A_{1g}$  vibration of the plane oxygen requires admixture of apical oxygens.

Another interesting resonance phenomenon is the appearance of Raman forbidden, ir-allowed phonons near resonance in the non-superconducting modification of the 123-materials (YBa<sub>2</sub>Cu<sub>3</sub>O<sub>6</sub>). These modes are shown by wavy arrows in Fig. 9. They are interpreted as quadrupole allowed, induced by the Fröhlich electron-phonon interaction.<sup>41</sup> The observed peaks have been assigned to the LO components of vibrations perpendicular to c ( $E_u$  symmetry), since their frequencies agree with ir-reflectivity data.<sup>42</sup> Of the six modes of this symmetry four are seen in Fig. 9. They correspond to vibrations of the yttrium (206 cm<sup>-1</sup>), OHI.III (266 cm<sup>-1</sup>, OIV (417 cm<sup>-1</sup>) and OII (645 cm<sup>-1</sup>).

The reason why no ir-active modes polarized along c  $(A_{2u})$  are observed is attributed to the large effective masses of electronic bands along this direction (the Fröhlich scattering amplitude is proportional to the difference of the inverse effective masses of participating valence and conduction bands along the polarization direction of the phonon<sup>43</sup>).



FIGURE 10 Mixed cfe-phonon coupled modes in the Raman spectra of NdBa<sub>2</sub>Cu<sub>3</sub>O<sub>7</sub> (<sup>16</sup>O and <sup>18</sup>O isotopes).<sup>21,22</sup>

### 5. MIXED PHONON CRYSTAL-FIELD EXCITATIONS

The rare earth ions of high- $T_C$  SC's have, with the exception of La and Y, strongly localized 4f electrons whose ionic levels are split by Coulomb interaction into many-electron angular momentum states. These states split further under the action of the rather low symmetry crystal field (e.g.  $D_{2h}$  for NdBa<sub>2</sub>Cu<sub>3</sub>O<sub>7</sub>,  $D_{4h}$  for the O<sub>6</sub> modification). They are at least doubly (Kramers) degenerate if the number of f-electrons is odd, such as for  $Nd^{3+}$  (3f-electrons). Transitions between the crystal field ground state and excited states (cfe) have been observed by neutron scattering.44,45 They are usually too weak to be seen in Raman scattering owing to the localized nature of the f-electrons. In some special cases, however, f-electron excitations are nearly degenerate with phonons of the same symmetry. Such is the case of NdBa<sub>2</sub>Cu<sub>3</sub>O<sub>7- $\delta$ </sub>. A  $B_{1g}$  (tetragonal notation) crystal field excitation is nearly degenerate with the  $B_{1q}$  phonon, whose frequency lies slightly above that of the cfe for O<sup>16</sup> and slightly below O<sup>18,21,22</sup> Electron-phonon coupling produces in this case a strong mixing of the  $B_{1g}$  electronic and vibronic excitations, a doublet results with nearly equal mixture of both components. Both members of the doublet can thus be seen in the Raman spectra (Fig. 10), with the higher frequency component slightly stronger than the lower one for O<sup>16</sup> and slightly weaker for O<sup>18</sup>, as expected. Both components shift by half of the change in the square root of the O-mass when  $O^{16}$  is replaced by  $O^{18}$ , owing to the fact that each component is only partially an O-vibration. With increasing temperature, the occupation of the crystal field levels changes, most consequential being the decrease of the population of the crystal field ground state. This results in an effective decrease of the coupling between phonons and cf levels<sup>21,41</sup> which is also displayed in Fig. 10.

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71

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