# Класичні обчислення

$0 \le a \le 2^n - 1$ $\qquad$ $B = \{0, 1\}$

$a$: $a_1 \dots a_n$, $b$: $b_1 \dots b_n$, $\dots$, $c$: $c_1 \dots c_n$ $\xrightarrow{f}$ $a'_1 \dots a'_n$, $b'_1 \dots b'_n$

$(a', b', \dots, c') = f(a, b, \dots, c)$

$f: B^m \to B^m$

$a \wedge b = a \text{ AND } b = z$

$a \vee b = a \text{ OR } b$

$\neg a = \text{NOT } a$

| a | b | z |
|---|---|---|
| 0 | 0 | 0 |
| 0 | 1 | 0 |
| 1 | 0 | 0 |
| 1 | 1 | 1 |

$f(b_1 \dots b_m) = (\neg b_1 \vee b_2 \vee \dots b_m) \wedge (b_1 \vee \neg b_2 \dots \vee b_m) \dots$

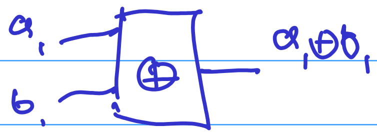
$a = a_1 \dots a_n$
$b = b_1 \dots b_n$

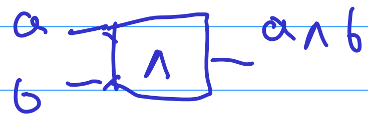
$f(a, b) = a + b$

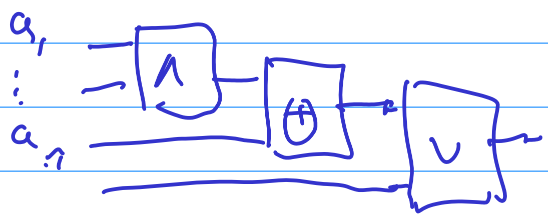
$f(a, b) = a \oplus b = (a_1 \oplus b_1)(a_2 \oplus b_2) \dots (a_n \oplus b_n)$

$\oplus$, XOR:

| a | b | z |
|---|---|---|
| 0 | 0 | 0 |
| 0 | 1 | 1 |
| 1 | 0 | 1 |
| 1 | 1 | 0 |




$f: B^n \to B^m$

$2^n \qquad 2^{m}$

$\underbrace{2^m \cdot \ldots \cdot 2^m}_{2^n} = (2^m)^{2^n}$

$a = a_1 \dots a_n$
$b = b_1 \dots b_n$

$f(a, b) = ab = z = z_1 \dots z_{2n}$ — поліном складність

$g(z) = a$, $a | z$, $a \neq 1$, $a \neq z$ — експ. скл. (не дов.)

Лінійна алгебра

$H = \mathbb{C}^n$ $\quad |v\rangle = \begin{pmatrix} \alpha_0 \\ \alpha_1 \\ \vdots \\ \alpha_{n-1} \end{pmatrix}$ $\quad \alpha_i \in \mathbb{C}$

$|v\rangle = \sum \alpha_i |i\rangle$ $\quad |0\rangle = \begin{pmatrix} 1 \\ 0 \\ \vdots \\ 0 \end{pmatrix} \dots \quad |i\rangle = \begin{pmatrix} 0 \\ \vdots \\ 1 \\ \vdots \\ 0 \end{pmatrix}$

$\langle v| := (|v\rangle)^{\dagger} = (\bar{\alpha}_0, \dots, \bar{\alpha}_{n-1})$ $\quad |w\rangle = \begin{pmatrix} \beta_0 \\ \vdots \\ \beta_{n-1} \end{pmatrix}$

$\langle v|w\rangle = \langle v| \cdot |w\rangle = \sum_i \bar{\alpha}_i \beta_i$ $\quad \langle v|w\rangle = \overline{\langle w|v\rangle}$

$\| |v\rangle \| = +\sqrt{\langle v|v\rangle} = \sqrt{\sum_i |\alpha_i|^2}$

$H = \mathbb{C}^2$ $\quad |+\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$ $\quad |-\rangle = \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)$

$\langle +|+\rangle = 1$ $\quad \langle -|-\rangle = 1$ $\quad \langle +|-\rangle = 0 = \langle -|+\rangle$

$A = \begin{pmatrix} a_{11} \dots a_{1n} \\ a_{n1} \dots a_{nn} \end{pmatrix}$ $\quad A|v\rangle$ $\quad A^{\dagger} = \bar{A}^{T} = \begin{pmatrix} \bar{a}_{11} \dots \bar{a}_{n1} \\ \vdots \\ \bar{a}_{1n} \dots \bar{a}_{nn} \end{pmatrix}$

$Pr_v = |v\rangle\langle v|$ $\quad Pr_v |w\rangle = |v\rangle \langle v| \cdot |w\rangle =$
$= \underbrace{\langle v|w\rangle}_{\in \mathbb{C}} \cdot |v\rangle \in \{ \alpha |v\rangle, \alpha \in \mathbb{C} \}$

$A = A^{\dagger}$ $\quad A = \sum_i \lambda_i |v_i\rangle\langle v_i|$ $\quad \lambda_i \in \mathbb{R}$

$\langle v_i|v_i\rangle = 1$

$A \geq 0 \iff \lambda_i \geq 0, \forall i$ $\quad \langle v_i|v_j\rangle = 0 \quad i \neq j$

$\forall |v\rangle, |w\rangle$

$U$ - унітар. $\quad |v'\rangle = U|v\rangle \quad \langle v'|w'\rangle = \langle v|w\rangle$

$|w'\rangle = U|w\rangle$

$UU^\dagger = I \quad \{U|0\rangle, \dots, U|n-1\rangle\}$ - орт. базис

$\{|b_0\rangle, \dots, |b_{n-1}\rangle\}, \quad \langle b_i|b_j\rangle = \delta_{ij}$

$U = \sum_i |b_i\rangle\langle i|$

Тензорні добутки $\quad H_1 \otimes H_2$

$H_1 = \mathbb{C}^{n_1} \quad H_2 = \mathbb{C}^{n_2}$

$\{|0\rangle \dots |n_1-1\rangle\} \quad \{|0\rangle \dots |n_2-1\rangle\} \quad \dim H_1 \otimes H_2 = n_1 n_2$

$\{|i\rangle \otimes |j\rangle\}^{n_1 n_2}. \quad |v\rangle = \sum_{ij} \alpha_{ij} |i\rangle \otimes |j\rangle$

$|v\rangle \otimes |w\rangle := \sum \alpha_i \beta_j |i\rangle \otimes |j\rangle$

$\sum \alpha_i |i\rangle \quad \sum \beta_j |j\rangle \quad \otimes : H_1 \times H_2 \to H_1 \otimes H_2$

$\exists |u\rangle \in H_1 \otimes H_2, \quad |u\rangle \neq |v\rangle \otimes |w\rangle$

$(A \otimes B)(|v\rangle \otimes |w\rangle) := A|v\rangle \otimes B|w\rangle$

$(A \otimes B)(\sum \alpha_{ij} |i\rangle \otimes |j\rangle) := \sum \alpha_{ij} A|i\rangle \otimes B|j\rangle$

$A = \begin{pmatrix} a_{11} & \dots & a_{1n_1} \\ \vdots & & \\ a_{n_1 1} & & a_{n_1 n_1} \end{pmatrix} \quad B = \begin{pmatrix} b_{11} & & b_{1n_2} \\ \vdots & & \\ b_{n_2 1} & & b_{n_2 n_2} \end{pmatrix}$

$n_1 n_2 \times n_1 n_2$

$A \otimes B = \begin{pmatrix} a_{11} B & \dots & a_{1n_1} B \\ \vdots & & \\ a_{n_1 1} B & \dots & a_{n_1 n_1} B \end{pmatrix} = \begin{pmatrix} a_{11} b_{11} & \dots & a_{11} b_{1n_2} & \dots \\ \vdots & \ddots & & \\ & & & \end{pmatrix}.$

$$|v\rangle = \begin{pmatrix} \alpha_0 \\ \alpha_1 \end{pmatrix}, \quad |w\rangle = \begin{pmatrix} \beta_0 \\ \beta_1 \end{pmatrix} \qquad |v\rangle \otimes |w\rangle = \begin{pmatrix} \alpha_0 \beta_0 \\ \alpha_0 \beta_1 \\ \alpha_1 \cdot \beta_0 \\ \alpha_1 \cdot \beta_1 \end{pmatrix}$$

$$(c_1 A_1 + c_2 A_2) \otimes B = c_1 \cdot A_1 \otimes B + c_2 A_2 \otimes B$$

$$(A \otimes B) \cdot (C \otimes D) = AC \otimes BD \qquad (A \otimes B)^\dagger = A^\dagger \otimes B^\dagger$$

$$\mathbb{C}^2 \otimes \mathbb{C}^2 \qquad |v\rangle = \underbrace{(|0\rangle + i|1\rangle)}_{|v_1\rangle} \otimes \underbrace{((1+i)|0\rangle + 2|1\rangle)}_{|v_2\rangle} =$$

$$= (1+i)|0\rangle \otimes |0\rangle + \ldots$$

$$\langle v|v\rangle = \langle v_1| \otimes \langle v_2| \cdot |v_1\rangle \otimes |v_2\rangle =$$

$$= \langle v_1| \cdot |v_1\rangle \otimes \langle v_2| \cdot |v_2\rangle = 2 \cdot 6 = 12$$

$$\langle v_1|v_1\rangle = (\langle 0| - i\langle 1|) \cdot (|0\rangle + i|1\rangle) = 1 + 1 = 2$$

$$\langle v_2|v_2\rangle = ((1-i)\langle 0| + 2\langle 1|)((1+i)|0\rangle + 2|1\rangle) = 2 + 4 = 6$$

$$A = \begin{pmatrix} a_{11} \ldots a_{1m} \\ \quad a_{nm} \end{pmatrix} \qquad Tr(A) = \sum a_{ii} \qquad E_{ij} = \begin{array}{c c} & j \\ i & \begin{pmatrix} 0 & \vdots & 0 \\ \ldots & 1 & \ldots \\ 0 & \vdots & 0 \end{pmatrix} \end{array}$$

$$A = \sum_{i,j} a_{ij} E_{ij} = \sum a_{ij} |i\rangle\langle j|$$

$$\{|b_0\rangle \ldots |b_{n-1}\rangle\} \qquad A = \sum a'_{ij} |b_i\rangle\langle b_j| \qquad A' = \begin{matrix} a'_{11} \ldots a'_{1m} \\ \ldots \end{matrix}$$

$$A' = \sum_{ij} a'_{ij} |i\rangle\langle j|$$

$$U|i\rangle = |b_i\rangle$$

$$UA'U^\dagger = \sum a'_{ij} U|i\rangle\langle j|U^\dagger = \sum a'_{ij} |b_i\rangle\langle b_j| = A$$

$(AB)^\dagger =$
$= B^\dagger \cdot A^\dagger$

$$\langle j| \cdot U^\dagger = (U|j\rangle)^\dagger \qquad UA'U^\dagger = A$$

$$Tr(AB) = Tr(BA) \qquad Tr(A_1 A_2 \ldots A_n) = Tr(A_2 \ldots A_n A_1)$$

31 $(a \oplus b) \wedge (a \vee \neg c) = z$

| a | b | c | z |
|---|---|---|---|
| 0 | 0 | 0 | ? |
| ⋮ | | | |
| 1 | 1 | 1 | |

32 $|v\rangle = (|0\rangle + (3-i)|1\rangle) \otimes ((2+i)|0\rangle + i|1\rangle) \in \mathbb{C}^2 \otimes \mathbb{C}^2$

$\langle v|v\rangle = ?$

33 $A = |1\rangle\langle -| + |+\rangle\langle 0|$

$|v\rangle = |0\rangle + i|1\rangle$

$$A|v\rangle = \underset{?}{\alpha_0}|0\rangle + \underset{?}{\alpha_1}|1\rangle$$