

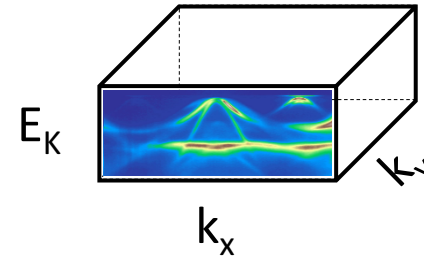
Курс: [Електронна структура та властивості низькорозмірних систем](#)

Лектор: Олександр Кордюк

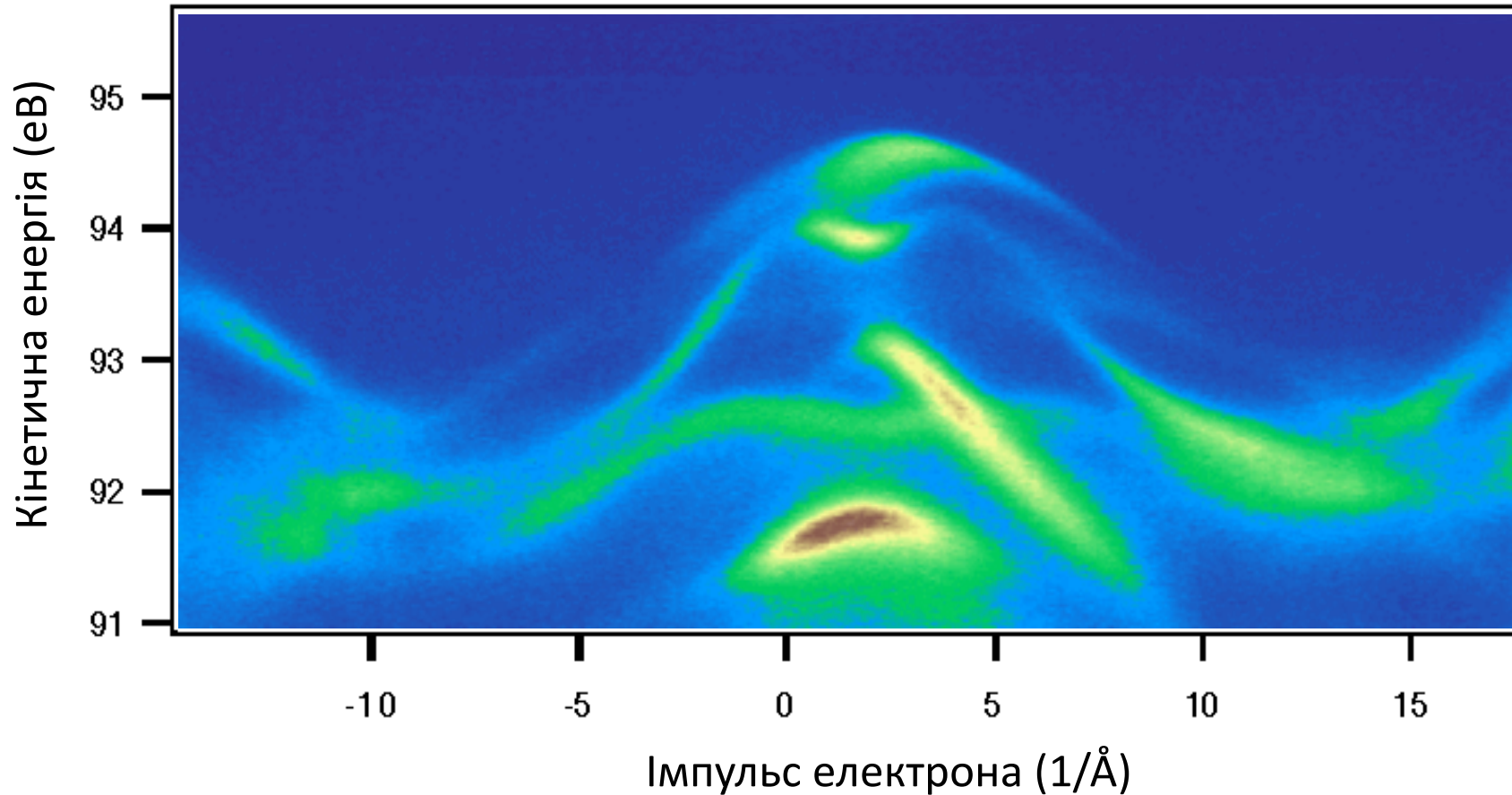
Лекції 8, 9: Спектральна функція

- Структура електронного спектру: дисперсія, розмиття, інтенсивність
- Симуляція спектральної функції
- Поняття квазічастинок
- Функція Гріна як геометрична прогресія
- Одночастункова спектральна функція та власна енергія
- Концепція Фермі-рідини Ландау
- Діаграми Файнмана
- Кореляція та двохчастинкова функція – функція Ліндгарда
- Електронна сприйнятливість та спектр спінових флуктуацій

Структура електронного спектру



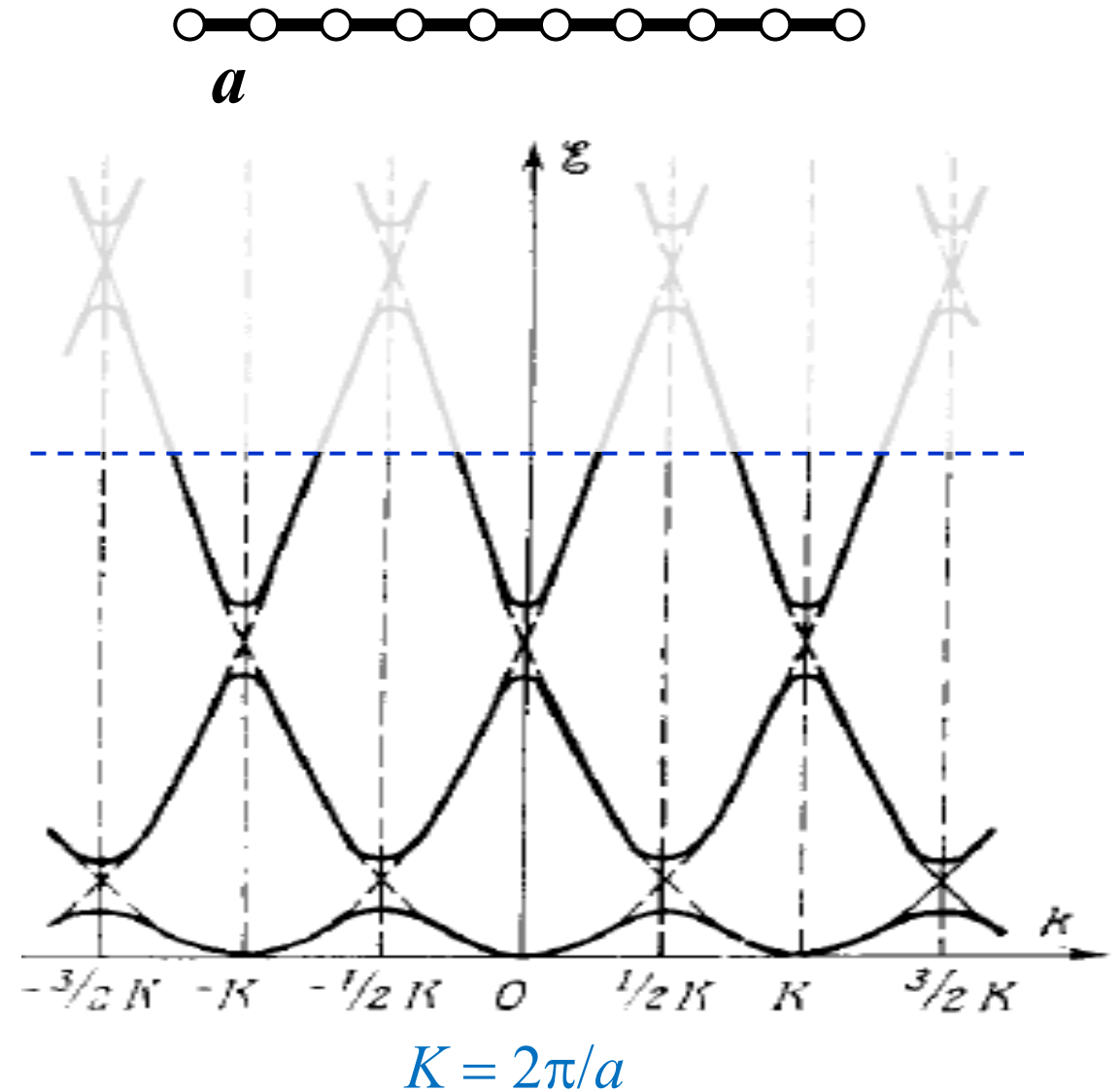
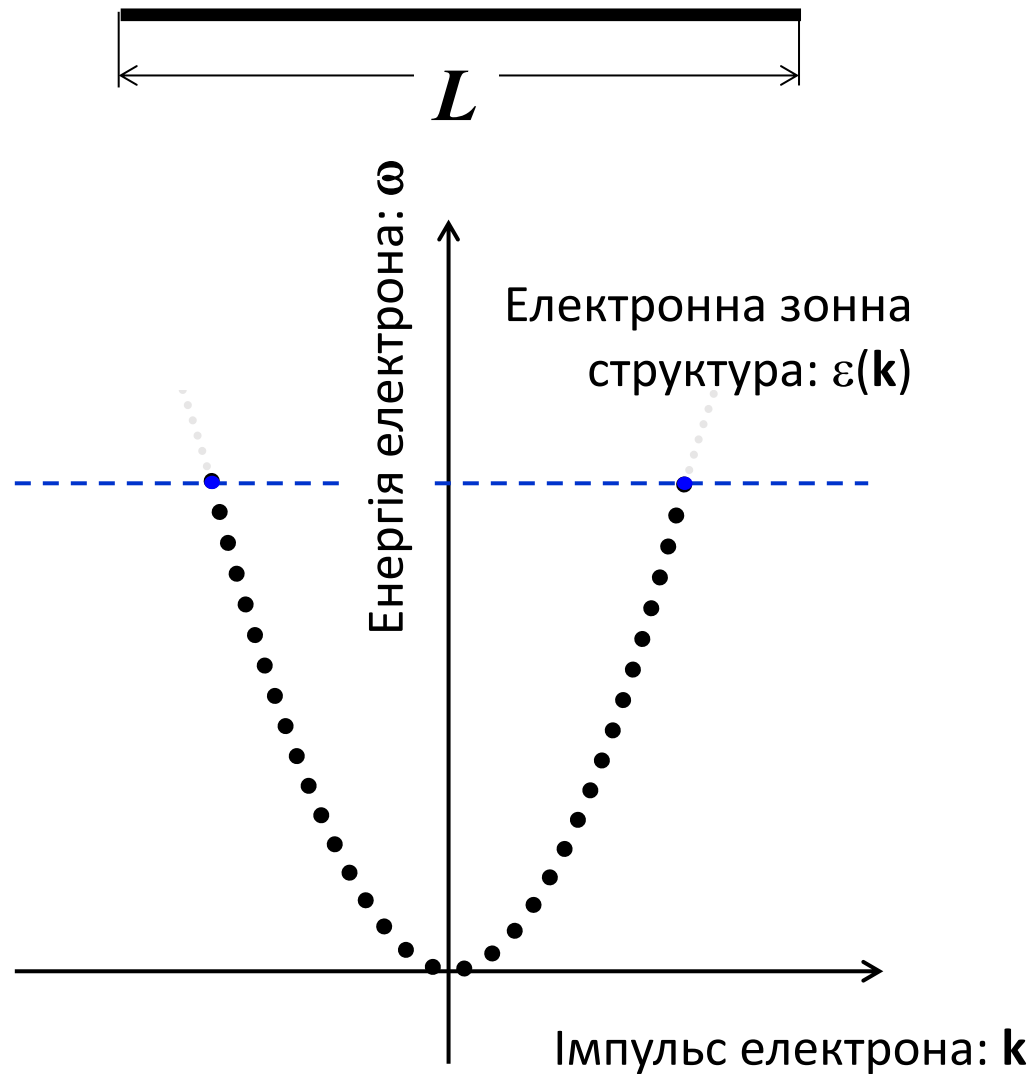
TiSe₂ - «ексітонний ізолятор»



Імовірність
знайти електрон
з імпульсом \mathbf{k}
та енергією ω

- дисперсія
- **розмиття**
- інтенсивність

Дисперсія блохівських електронів / інтерференція перескоків - наслідок когерентної взаємодії електронів з кристалом



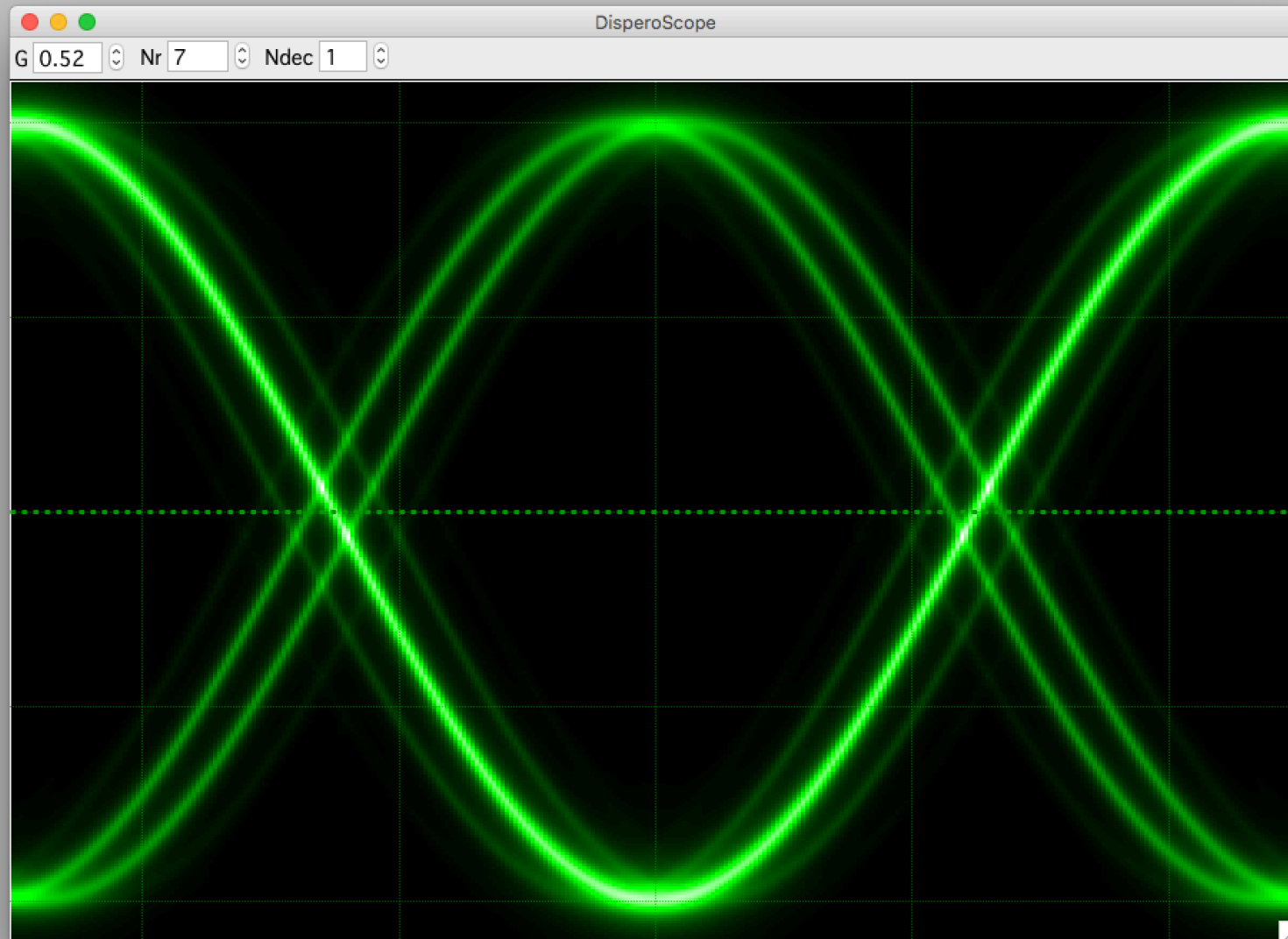
Spectral function simulation

$$A(k, \omega) =$$

$$= \sum_{i=-(Nr-1)}^{Nr-1} \frac{\exp(-Di)}{(\omega - \varepsilon_i(k))^2 + \delta^2}$$

$$\varepsilon_i(k) = -\cos[2\pi(x + Gi)]$$

$$\delta = 0.03$$



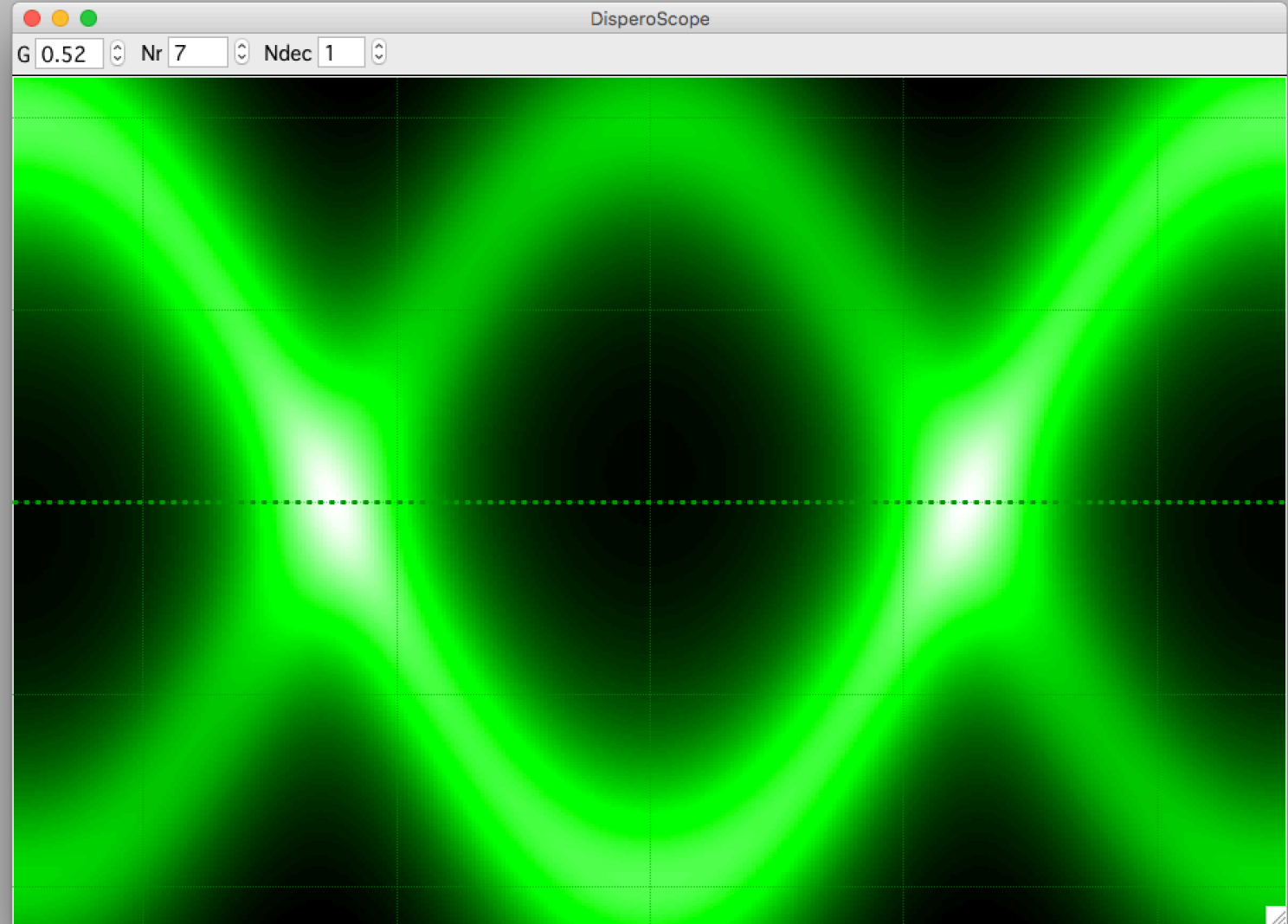
Spectral function simulation

$$A(k, \omega) =$$

$$= \sum_{i=-(Nr-1)}^{Nr-1} \frac{\exp(-Di)}{(\omega - \varepsilon_i(k))^2 + \delta^2}$$

$$\varepsilon_i(k) = -\cos[2\pi(x + Gi)]$$

$$\delta = 0.3$$



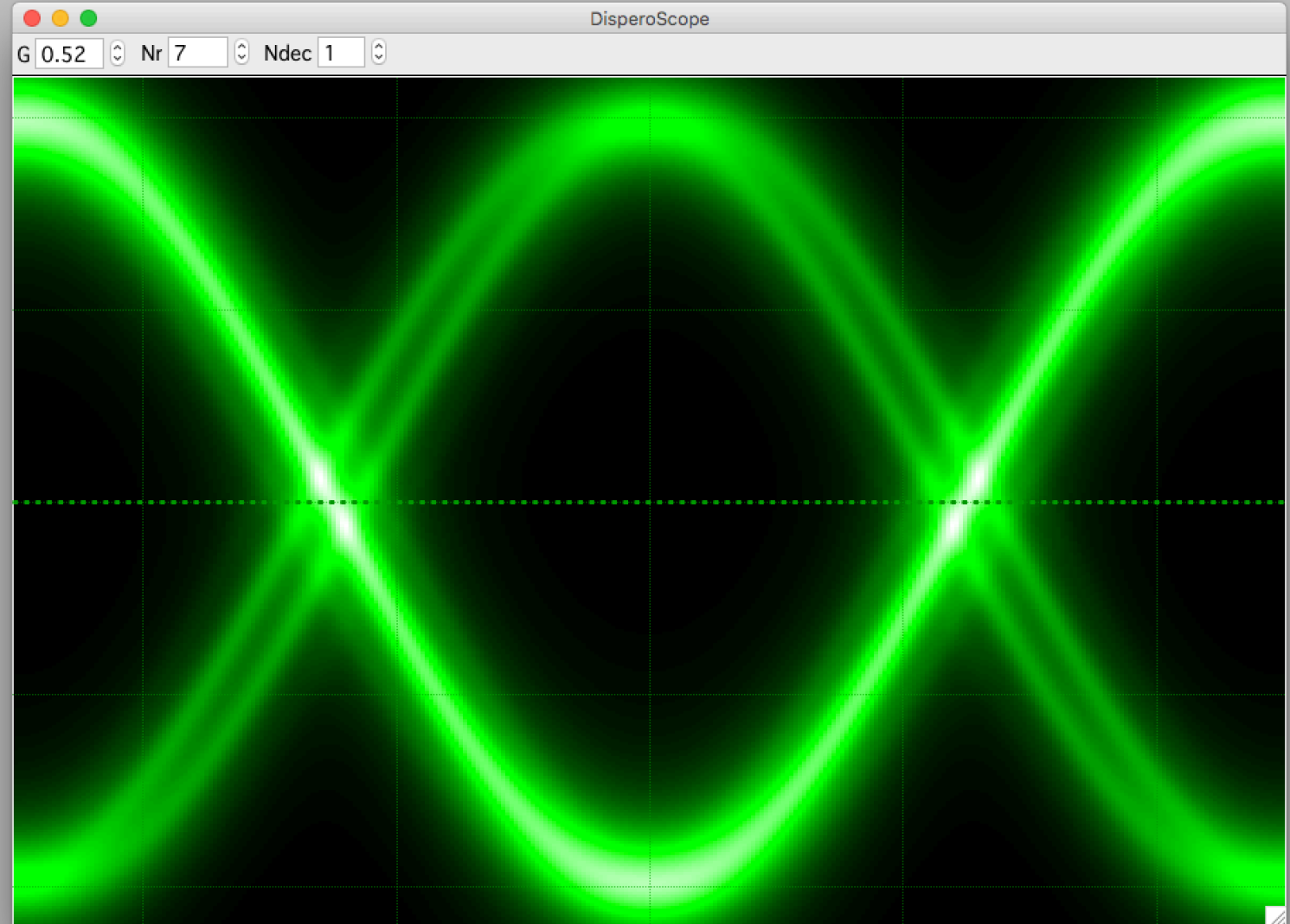
Spectral function simulation

$$A(k, \omega) =$$

$$= \sum_{i=-(Nr-1)}^{Nr-1} \frac{\exp(-Di)}{(\omega - \varepsilon_i(k))^2 + \delta^2}$$

$$\varepsilon_i(k) = -\cos[2\pi(x + Gi)]$$

$$\delta = 0.1$$



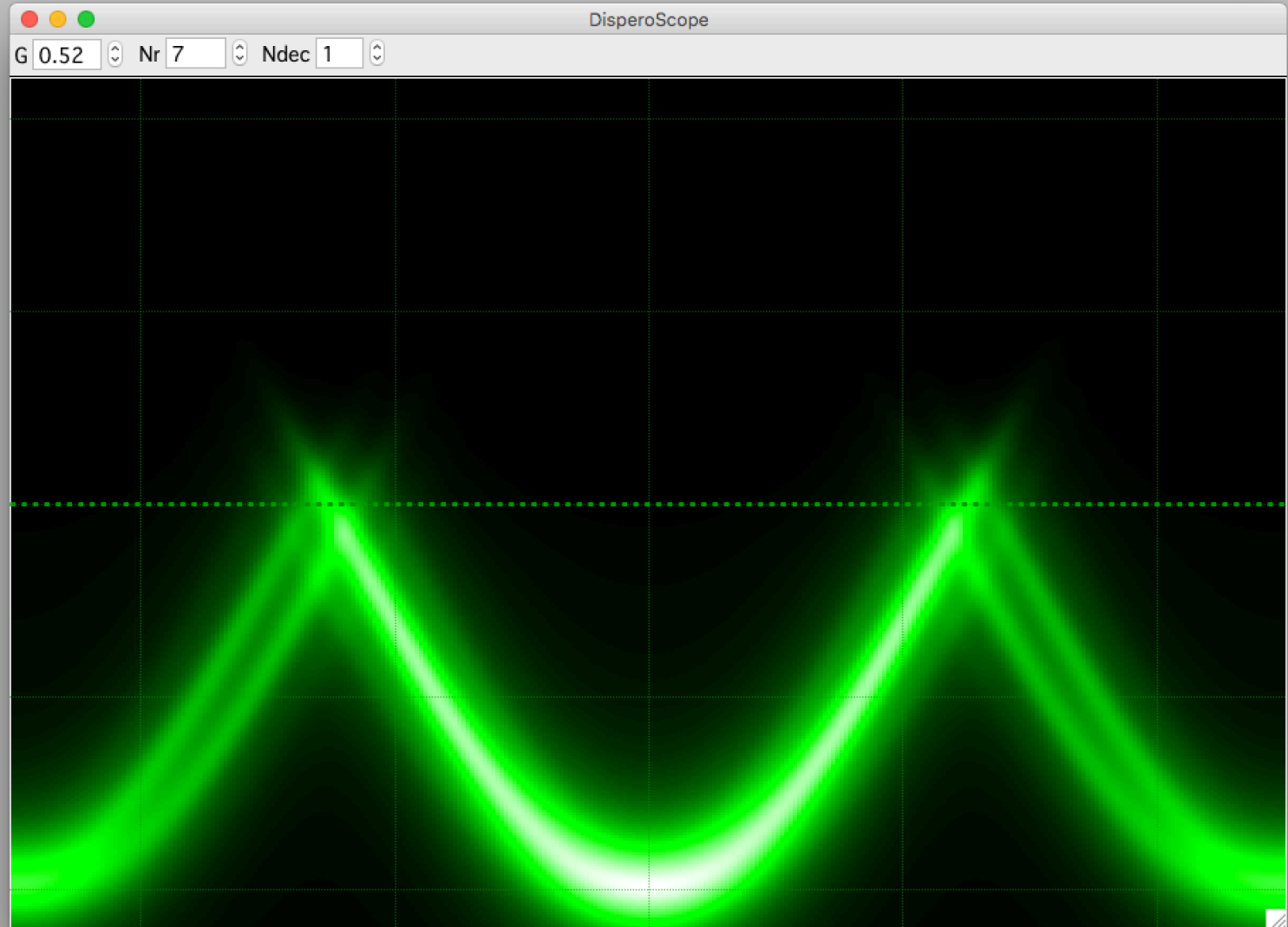
Spectral function simulation

$$A(k, \omega)f(\omega) =$$

$$= \sum_{i=-(Nr-1)}^{Nr-1} \frac{\exp(-Di)}{(\omega - \varepsilon_i(k))^2 + \delta^2} \times \\ \times \frac{1}{1 + \exp(\omega/\delta)}$$

$$\varepsilon_i(k) = -\cos[2\pi(x + Gi)]$$

$$\delta = 0.1$$



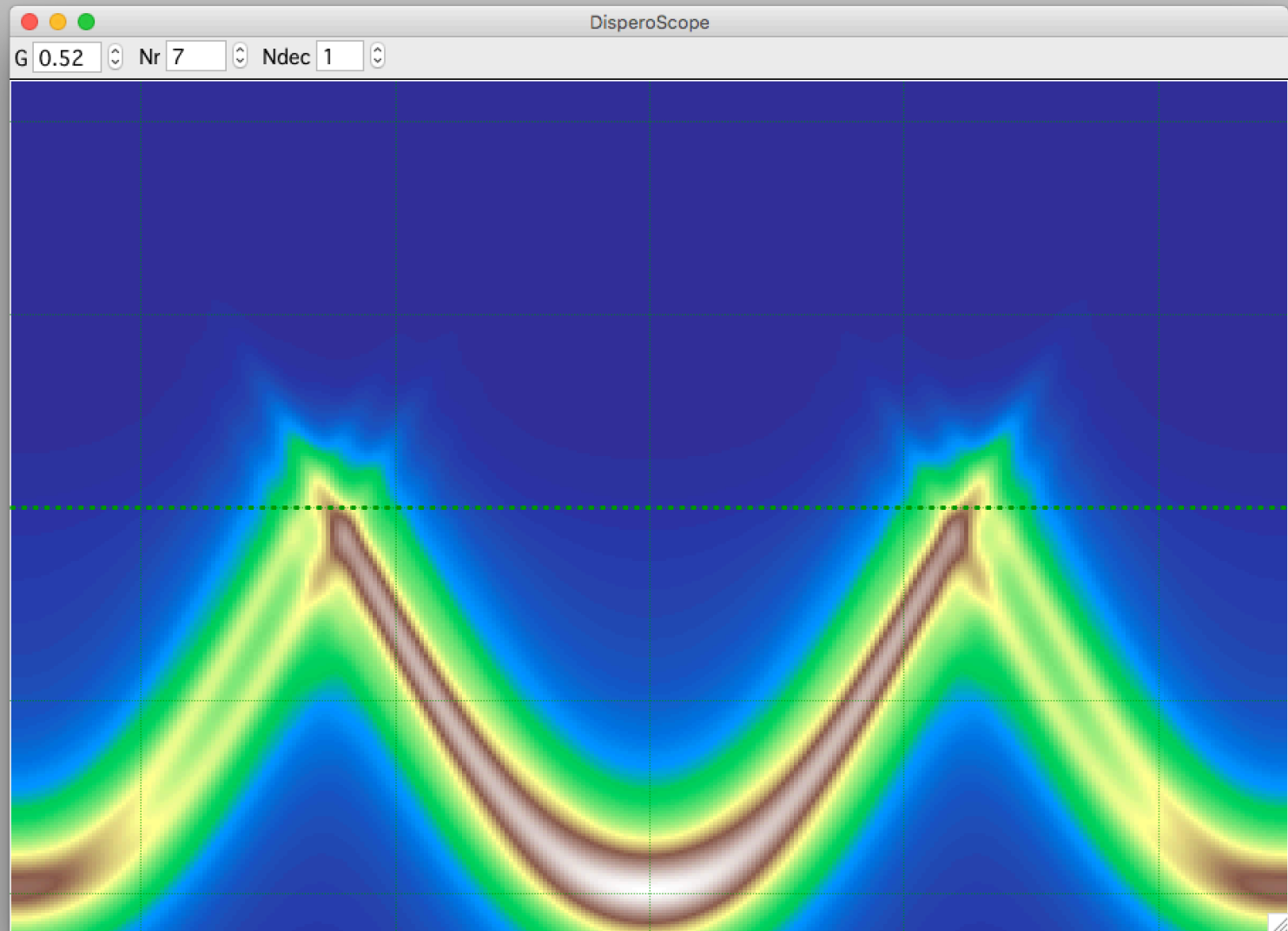
Spectral function simulation

$$A(k, \omega)f(\omega) =$$

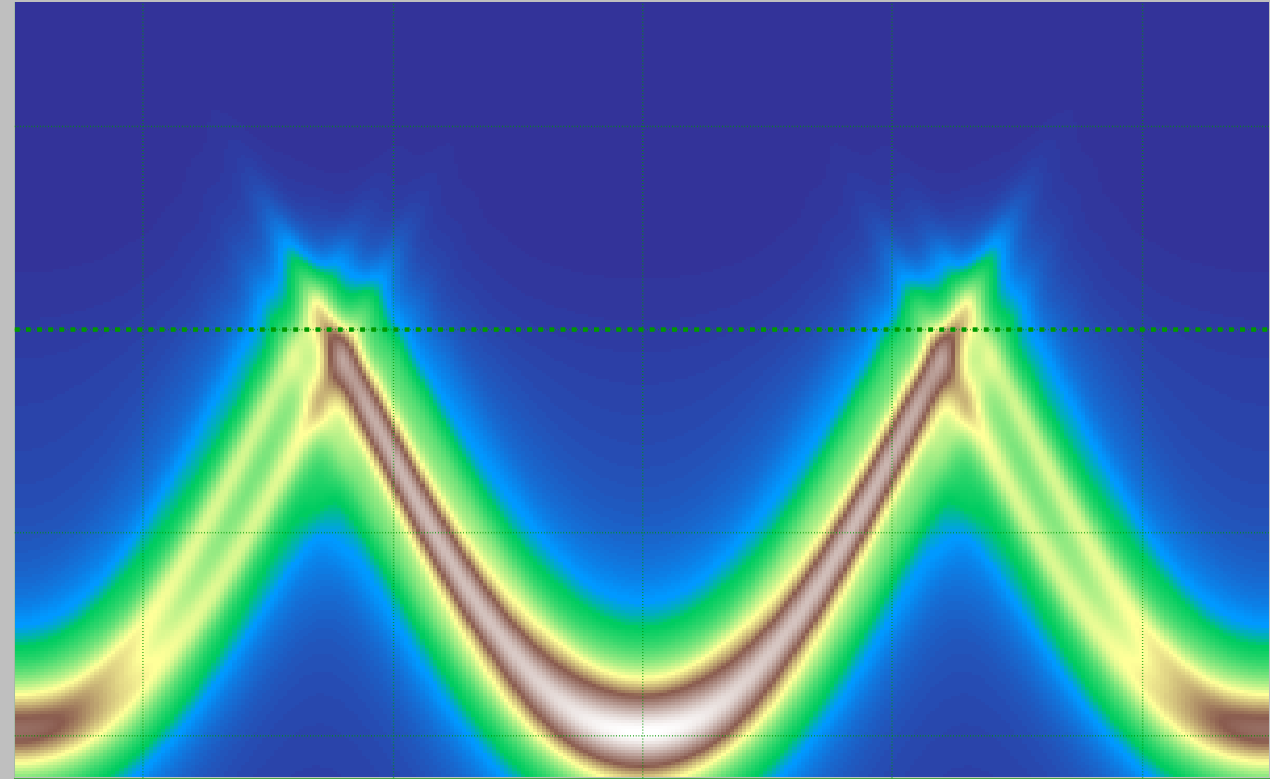
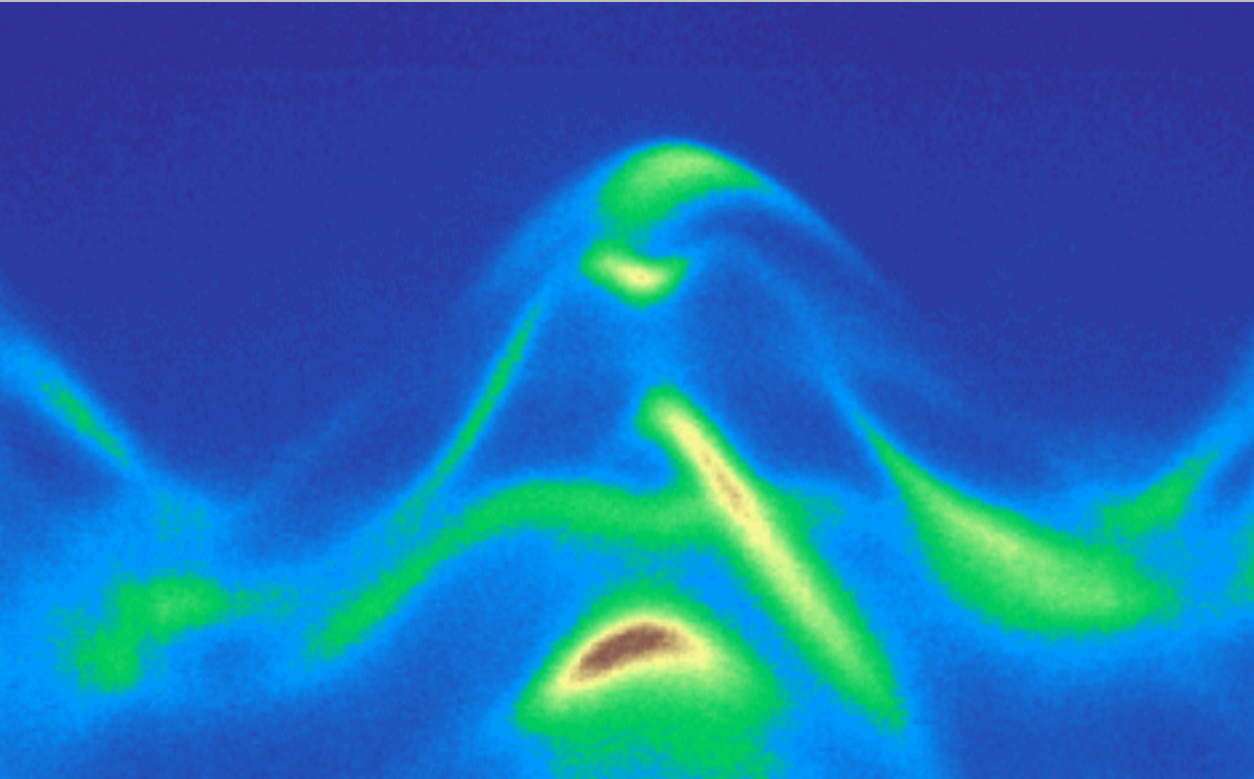
$$= \sum_{i=-(Nr-1)}^{Nr-1} \frac{\exp(-Di)}{(\omega - \varepsilon_i(k))^2 + \delta^2} \times \\ \times \frac{1}{1 + \exp(\omega/\delta)}$$

$$\varepsilon_i(k) = -\cos[2\pi(x + Gi)]$$

$$\delta = 0.1$$



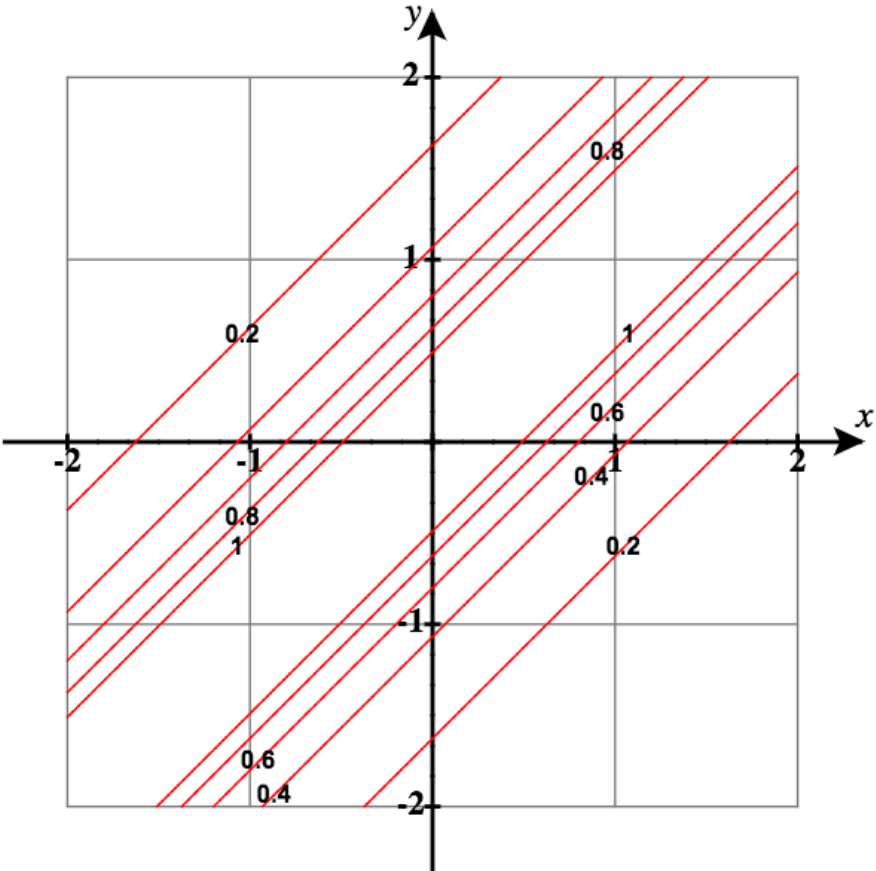
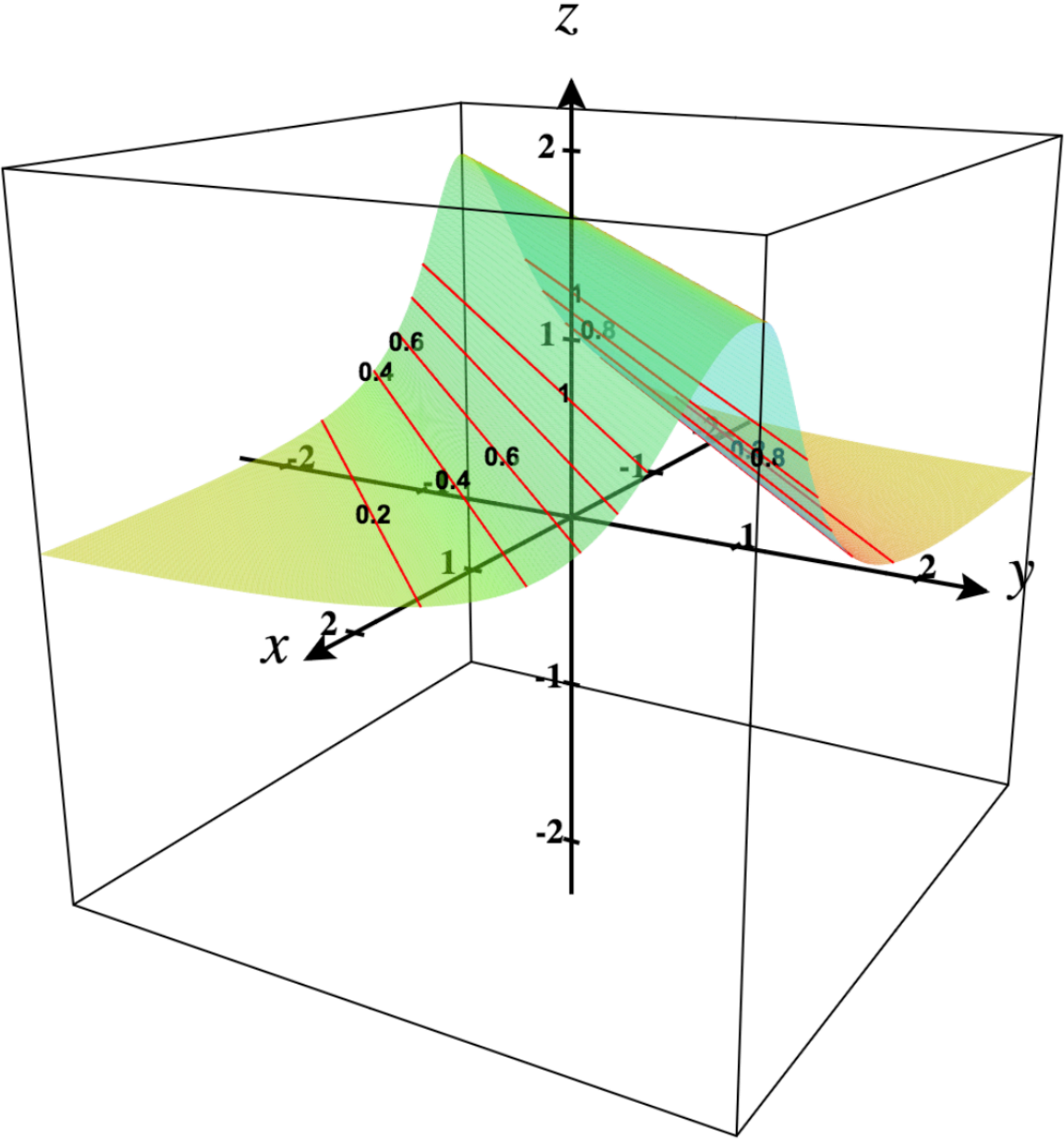
Spectral function simulation



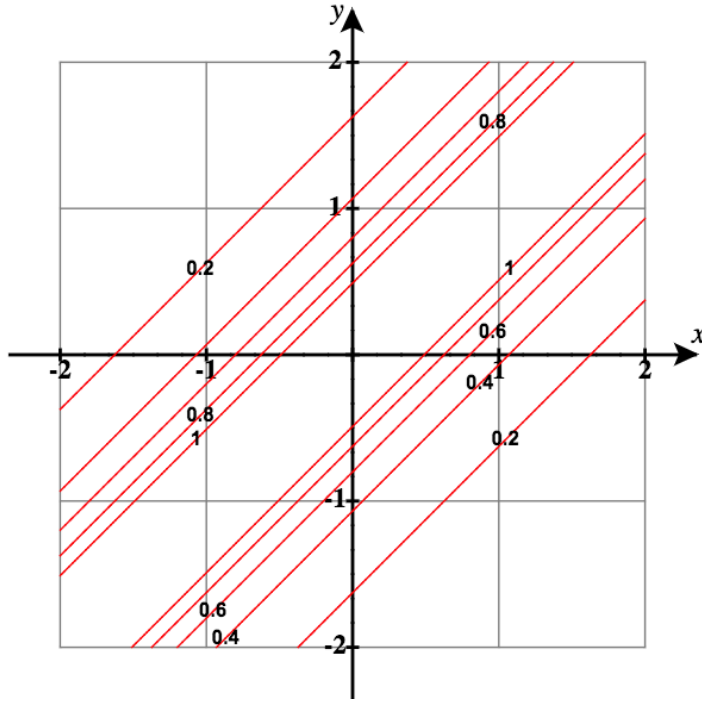
$$M(\mathbf{k}, n, h\nu) A(\mathbf{k}, \omega) f(\omega) = M(\mathbf{k}, n, h\nu) \frac{\delta(\omega)}{(\omega - \varepsilon_i(\mathbf{k}))^2 + \delta(\omega)^2} \frac{1}{1 + \exp(\omega/kT)}$$

Lorentzian

$$z = \frac{\delta}{(x - y)^2 + \delta^2}$$



Lorentzian – Spectral function



$$Z = \frac{\delta}{(x - y)^2 + \delta^2}$$

$$A(\omega, \mathbf{k}) = -\frac{1}{\pi} \frac{\Sigma''(\omega)}{(\omega - \varepsilon(\mathbf{k}) - \Sigma'(\omega))^2 + \Sigma''(\omega)^2}$$

Одно-частинкова спектральна функція = Im G

$$G_0(\omega, \mathbf{k}) = \frac{1}{\omega - \varepsilon(\mathbf{k}) + i\delta}$$

$$G(\omega, \mathbf{k}) = \frac{1}{\omega - \varepsilon(\mathbf{k}) - \Sigma(\omega)}$$

$$A(\omega, \mathbf{k}) = -\frac{1}{\pi} \text{Im } G(\omega, \mathbf{k})$$

$$A(\omega, \mathbf{k}) = -\frac{1}{\pi} \frac{\Sigma''(\omega)}{(\omega - \varepsilon(\mathbf{k}) - \Sigma'(\omega))^2 + \Sigma''(\omega)^2}$$

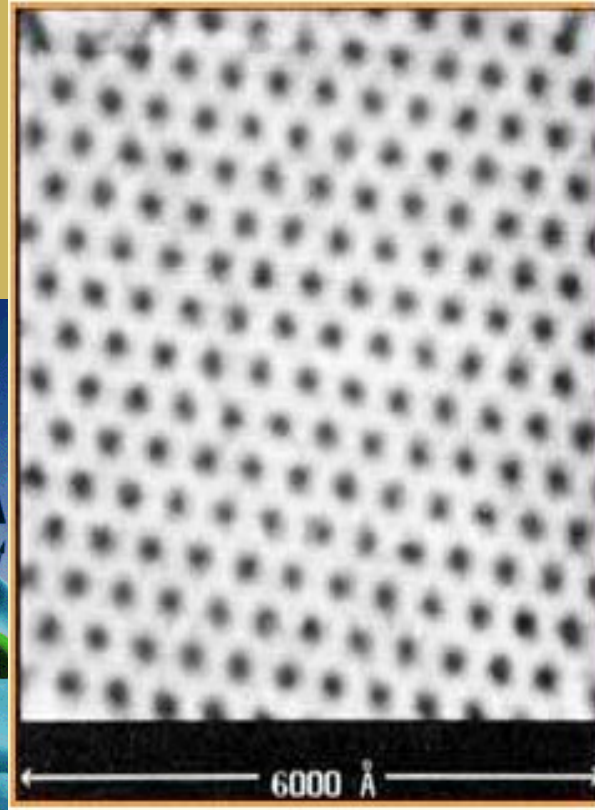
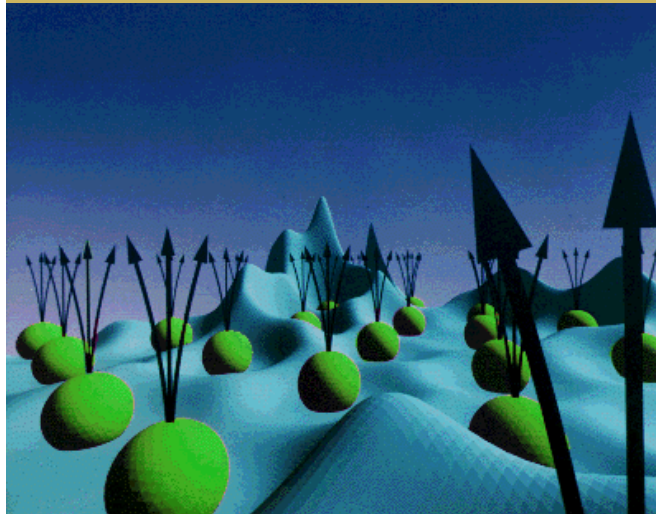
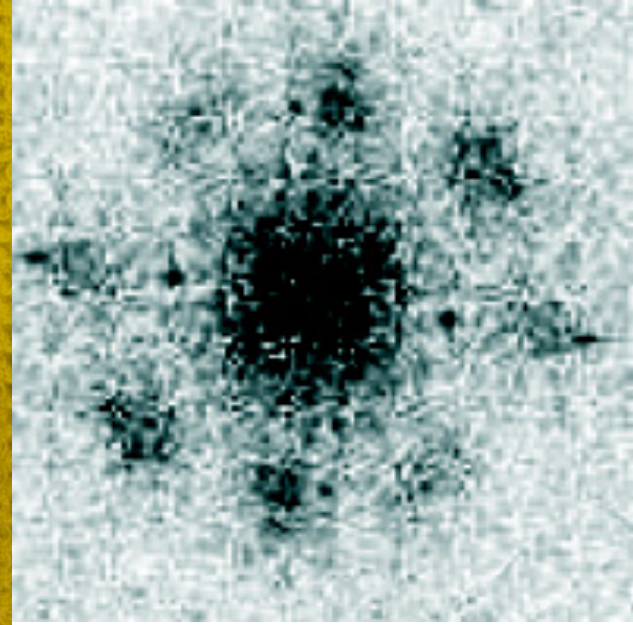
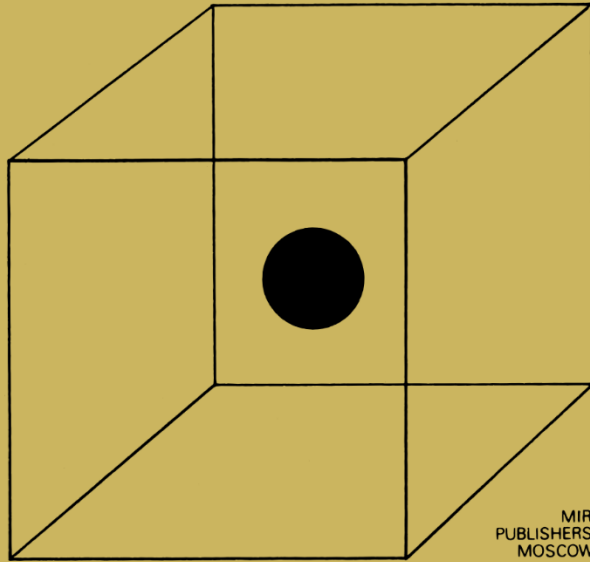
$$(G(\mathbf{k}, \omega))^{-1} = (G^0(\mathbf{k}, \omega))^{-1} - \Sigma(\mathbf{k}, \omega)$$

Quasiparticles / Квазічастинки

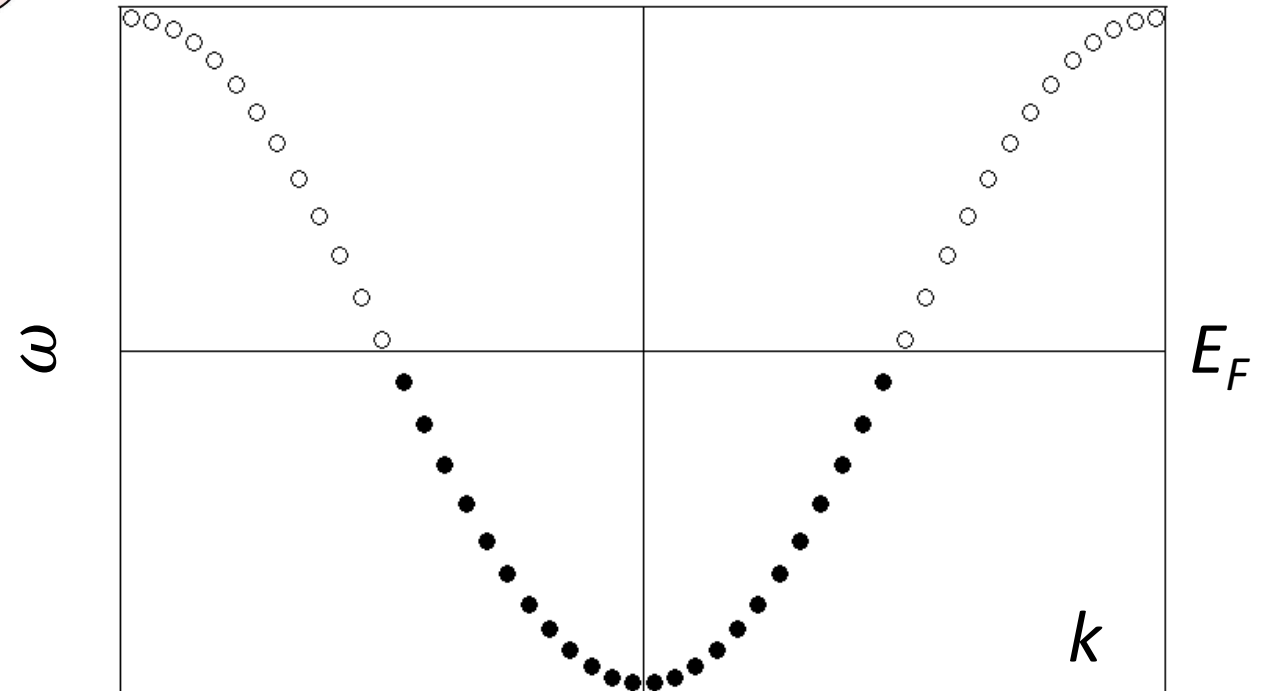
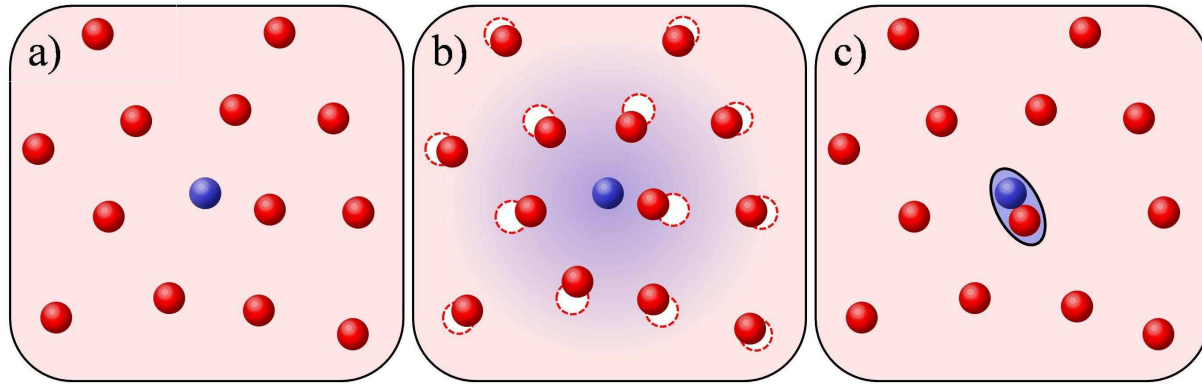
- i. Концепція квазічастинок - час життя та ефективна маса
- ii. Фермі-рідина
- iii. Функція Гріна, власна енергія
- iv. КК

M.I. Kaganov and I.M. Lifshits

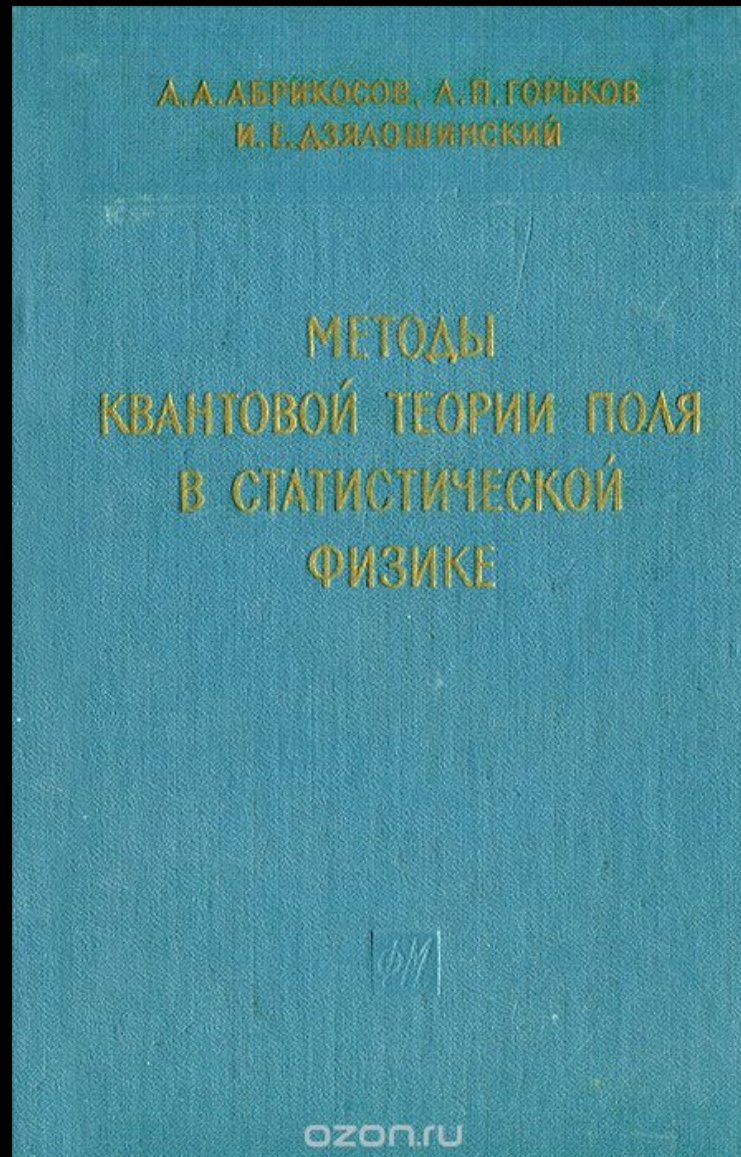
QUASI- PARTICLES



Квазічастинки



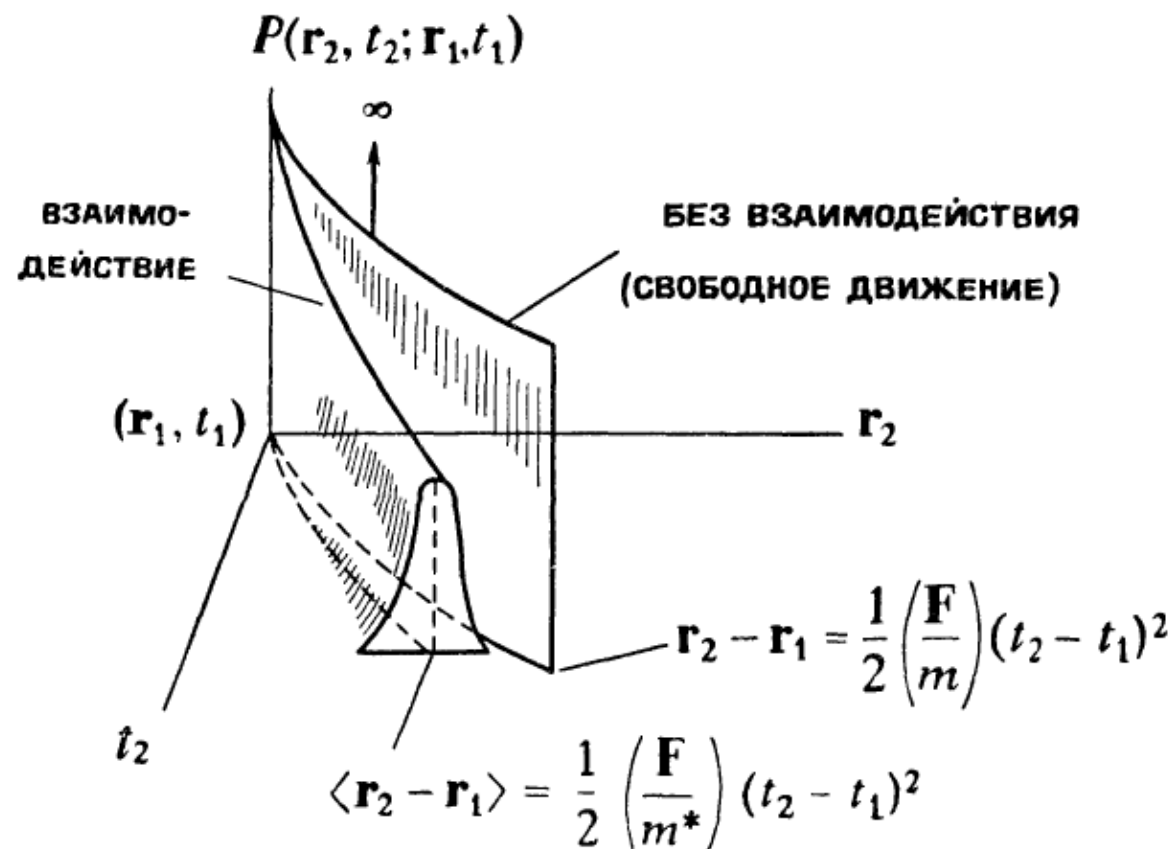
Спектральна функція = Im (функція Гріна)



$$\mathbf{r}_2 - \mathbf{r}_1 = \frac{1}{2} \left(\frac{\mathbf{F}}{m} \right) (t_2 - t_1)^2$$

$$P_0(\mathbf{r}_2, t_2; \mathbf{r}_1, t_1) = \delta \left[(\mathbf{r}_2 - \mathbf{r}_1) - \frac{1}{2} \left(\frac{\mathbf{F}}{m} \right) (t_2 - t_1)^2 \right]$$

$P(\mathbf{r}_2, t_2; \mathbf{r}_1, t_1)$ = Плотность вероятности (вероятность на единицу объема) того, что если частица в момент t_1 помещена в данную систему в точку \mathbf{r}_1 то она будет найдена в точке \mathbf{r}_2 в более поздний момент времени t_2 .



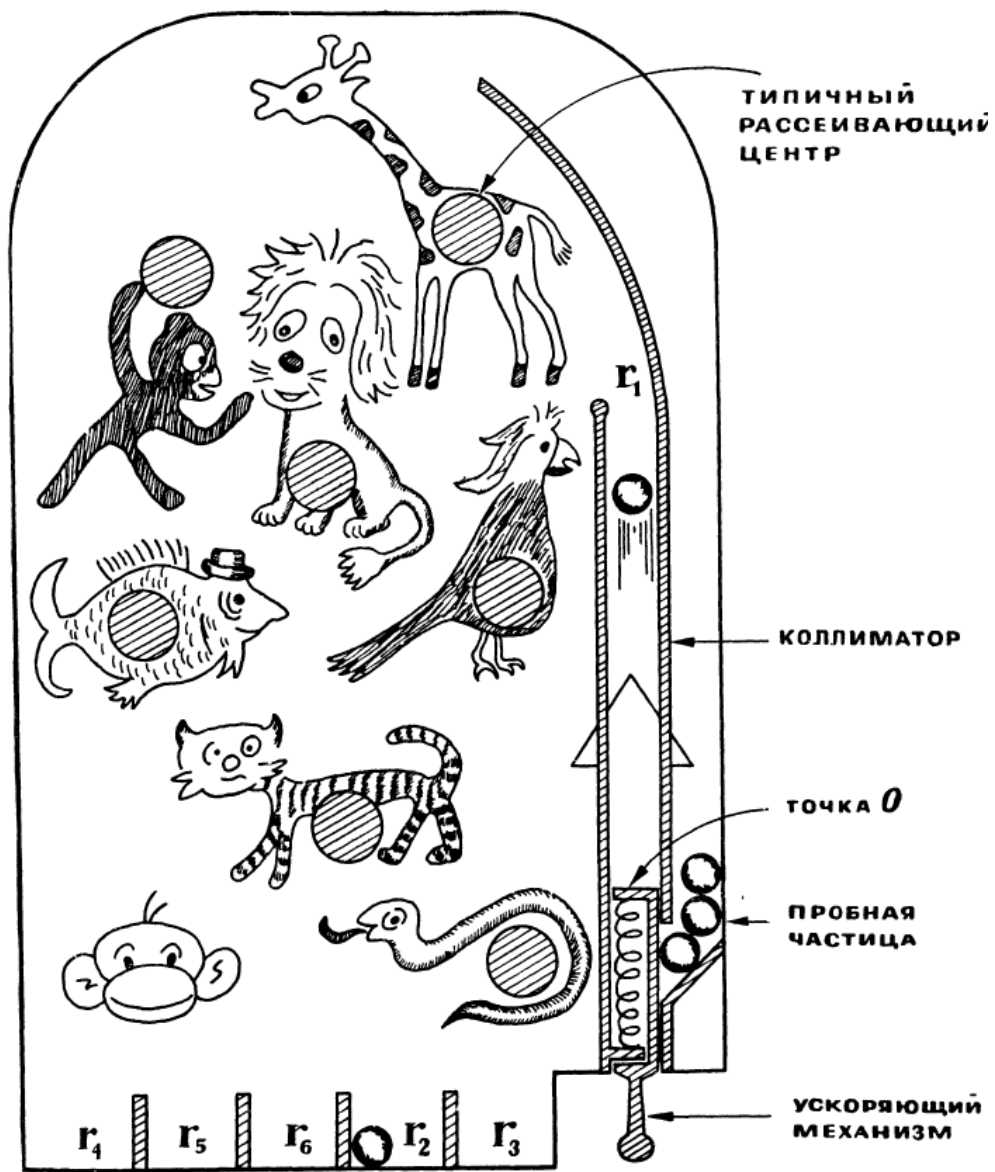
$$\langle \mathbf{r}_2 - \mathbf{r}_1 \rangle = \frac{1}{2} \left(\frac{\mathbf{F}}{m^*} \right) (t_2 - t_1)^2 \quad \text{для максимального значения } P$$

$$P_{\text{макс}}(\mathbf{r}_2, t_2; \mathbf{r}_1, t_1) \sim e^{-(t_2 - t_1)/\tau}$$

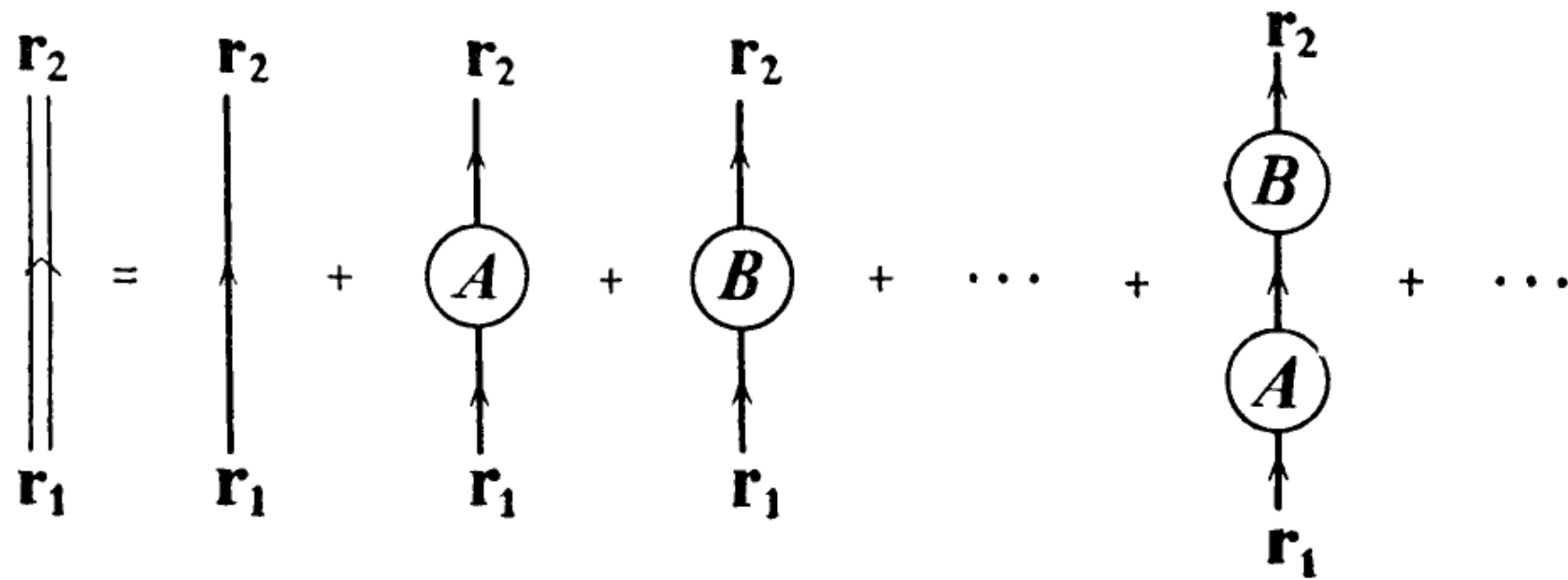
Р. Маттук

ФЕЙНМАНОВСКИЕ ДИАГРАММЫ
В ПРОБЛЕМЕ МНОГИХ ТЕЛ

ОZON.RU



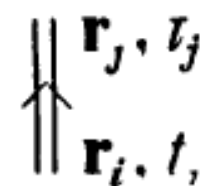
$$\begin{aligned}
 P(\mathbf{r}_2, \mathbf{r}_1) &= P_0(\mathbf{r}_2, \mathbf{r}_1) + P_0(\mathbf{r}_A, \mathbf{r}_1) P(A) P_0(\mathbf{r}_2, \mathbf{r}_A) + \\
 &\quad + P_0(\mathbf{r}_B, \mathbf{r}_1) P(B) P_0(\mathbf{r}_2, \mathbf{r}_B) + \dots \\
 &\quad + P_0(\mathbf{r}_A, \mathbf{r}_1) P(A) P_0(\mathbf{r}_B, \mathbf{r}_A) P(B) P_0(\mathbf{r}_2, \mathbf{r}_B) + \dots
 \end{aligned}$$



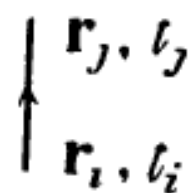
СЛОВО

Диаграмма

$$P(r_j, r_i, t_j - t_i)$$

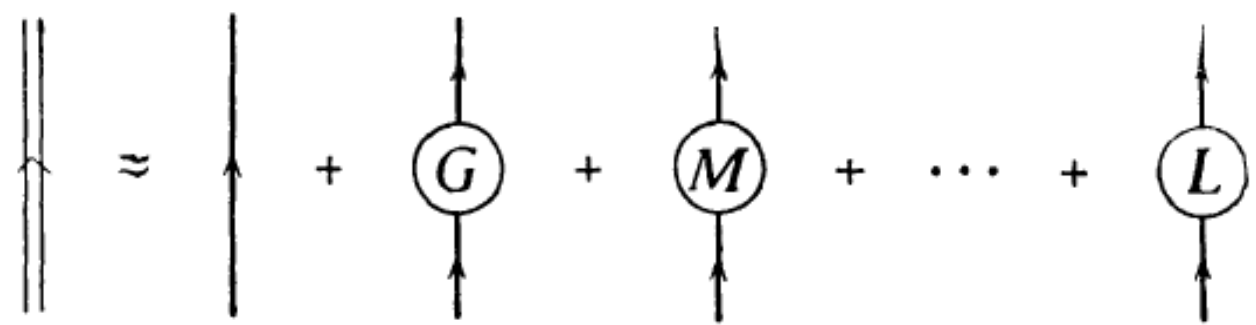
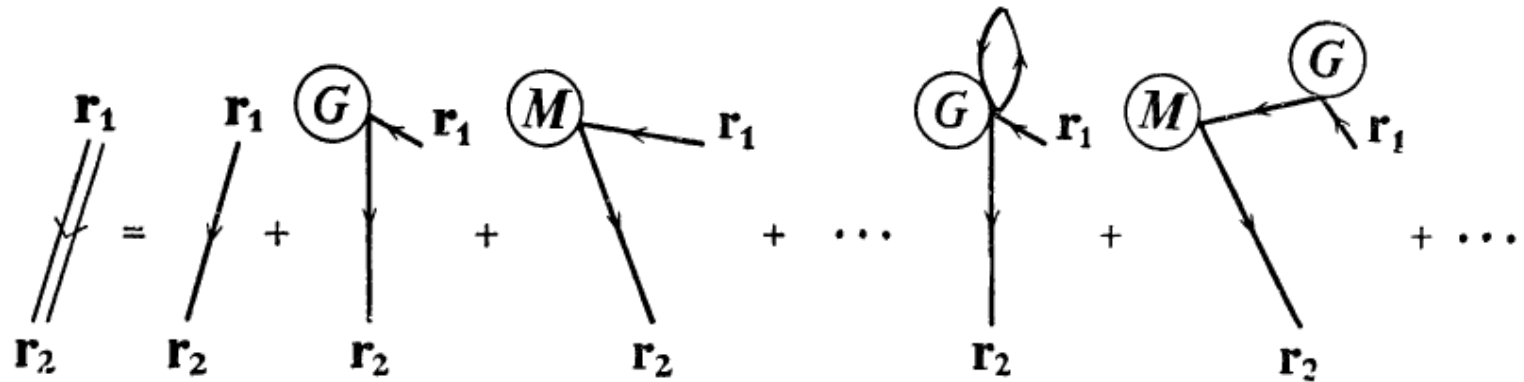
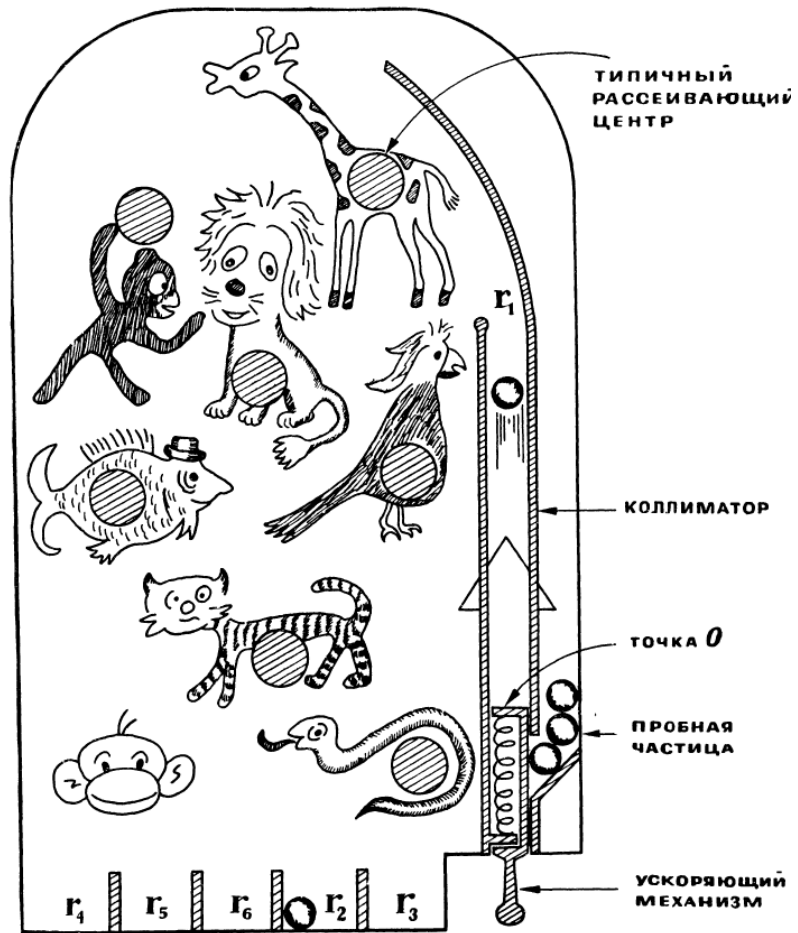


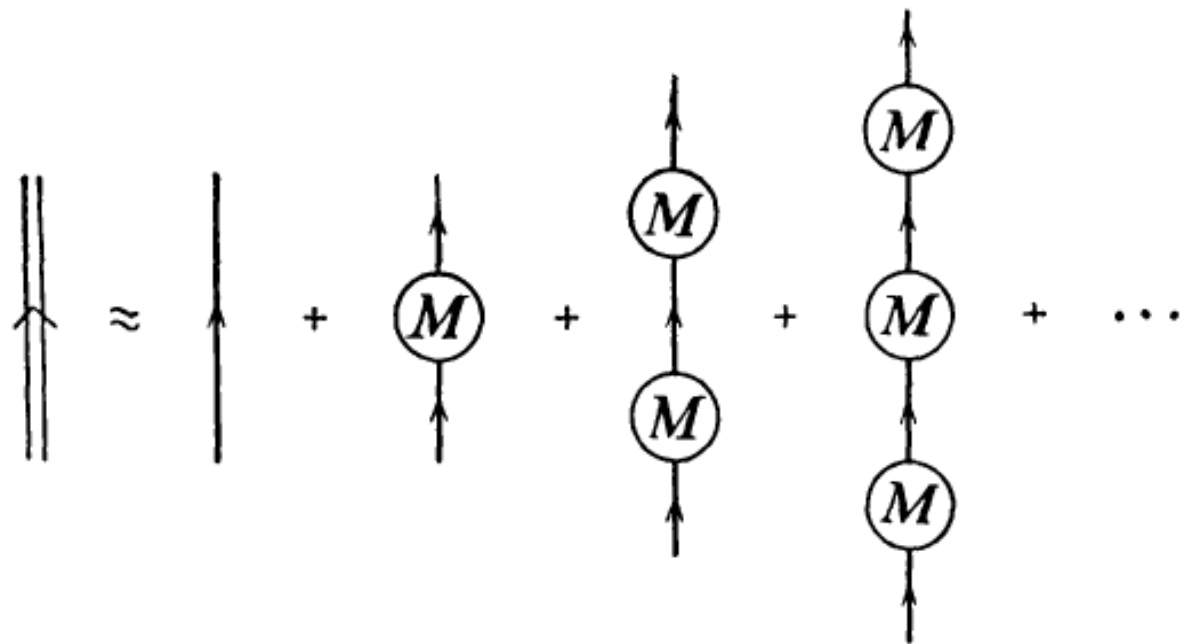
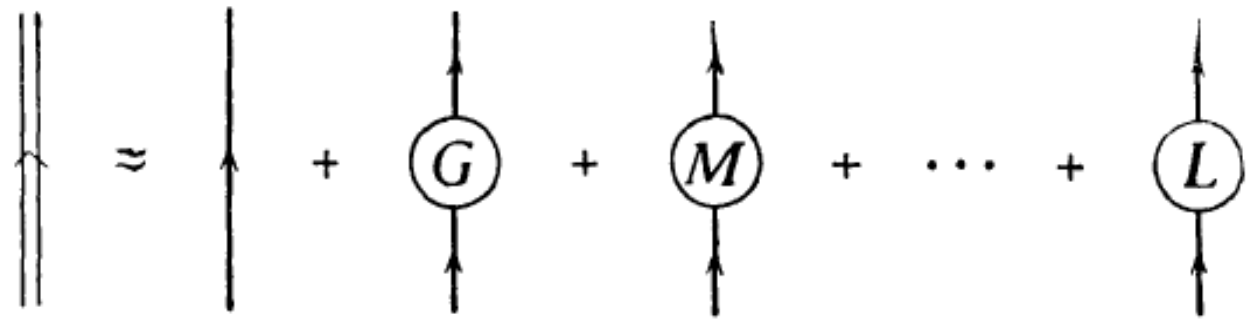
$$P_0(r_j, r_i, t_j - t_i)$$



$$P(A)$$







$$\uparrow\uparrow \approx \uparrow + \uparrow \times \uparrow \circlearrowleft M + \uparrow \times \uparrow^2 \circlearrowleft M + \uparrow \times \uparrow^3 \circlearrowleft M + \dots =$$

$$= \uparrow \times \left[1 + \uparrow \circlearrowleft M + \uparrow^2 \circlearrowleft M + \dots \right] =$$

$$= \frac{\uparrow}{1 - \uparrow \circlearrowleft M} = \frac{\uparrow}{1 - \uparrow \circlearrowleft M \times \uparrow} = \frac{1}{\uparrow^{-1} - \uparrow \circlearrowleft M}$$

$$S = a + aq + aq^2 + aq^3 + \dots = a/(1 - q)$$

Рівняння Дайсона

$$G^+(\mathbf{k}, \omega) \approx \frac{1}{(G_0^+)^{-1} - V_{M_{kk}}}$$

$$(G(\mathbf{k}, \omega))^{-1} = (G^0(\mathbf{k}, \omega))^{-1} - \Sigma(\mathbf{k}, \omega)$$

Одно-частинкова спектральна функція = Im G

$$(G(\mathbf{k}, \omega))^{-1} = (G^0(\mathbf{k}, \omega))^{-1} - \Sigma(\mathbf{k}, \omega)$$

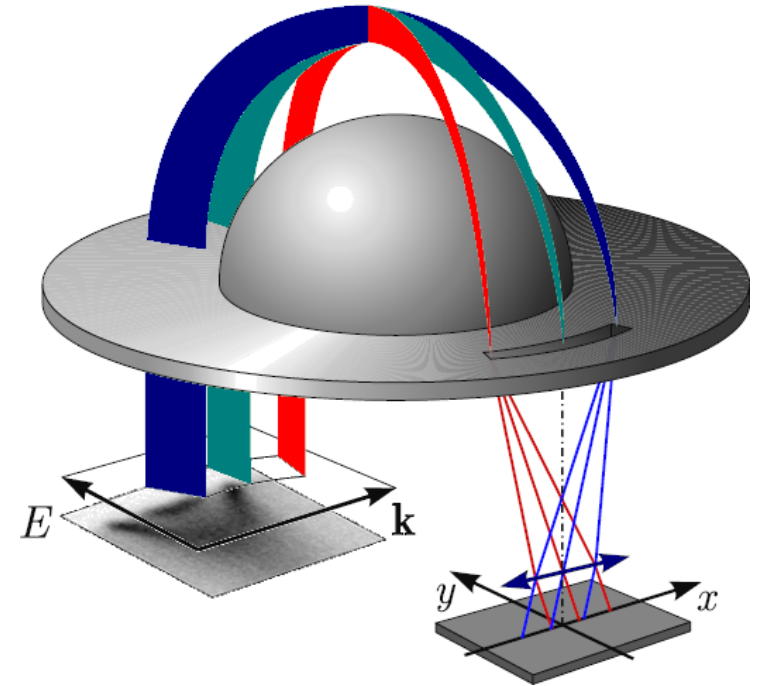
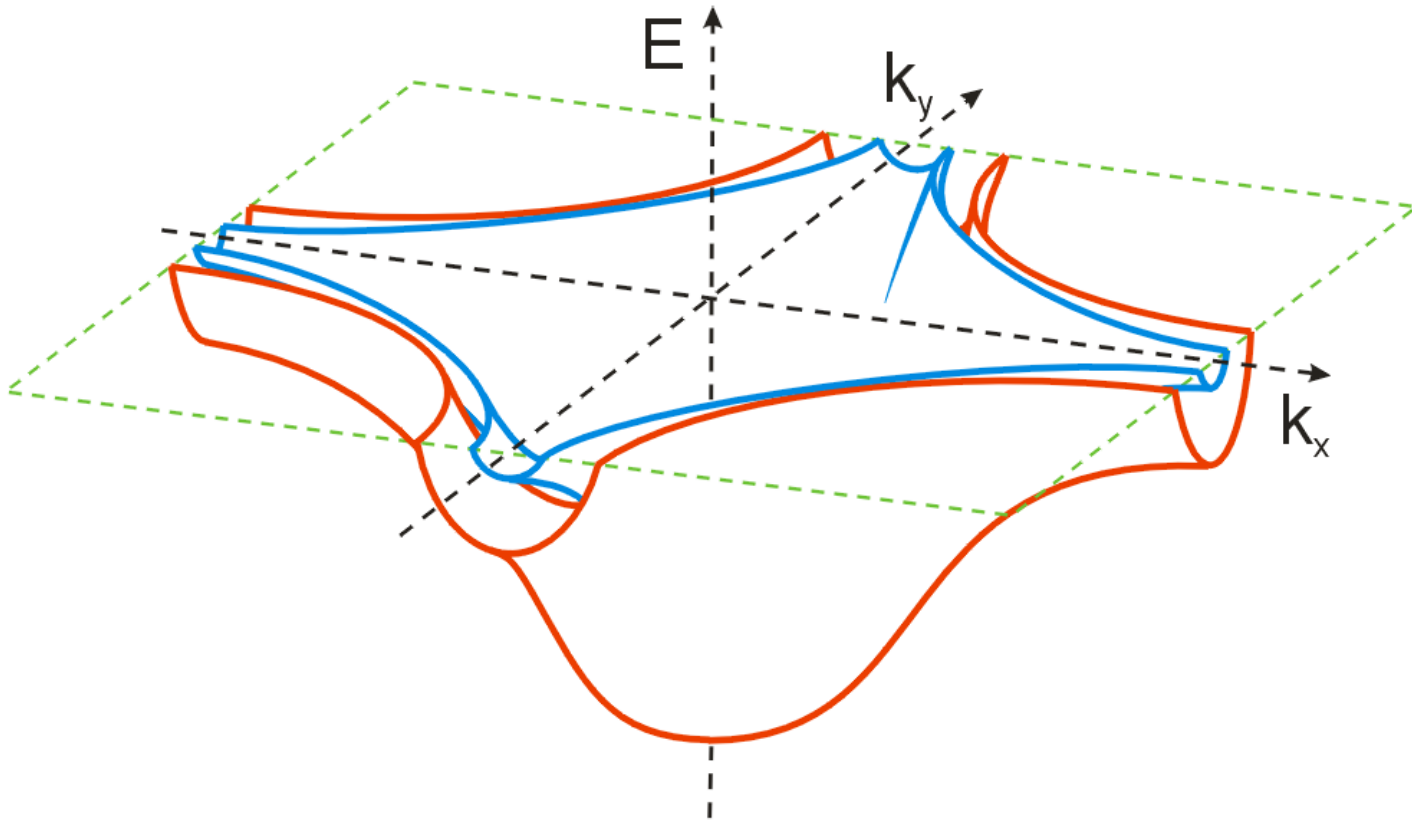
$$G_0(\omega, \mathbf{k}) = \frac{1}{\omega - \varepsilon(\mathbf{k}) + i\delta} \quad G(\omega, \mathbf{k}) = \frac{1}{\omega - \varepsilon(\mathbf{k}) - \Sigma(\omega)}$$

$$A(\omega, \mathbf{k}) = -\frac{1}{\pi} \text{Im } G(\omega, \mathbf{k})$$

$$A(\omega, \mathbf{k}) = -\frac{1}{\pi} \frac{\Sigma''(\omega)}{(\omega - \varepsilon(\mathbf{k}) - \Sigma'(\omega))^2 + \Sigma''(\omega)^2}$$

Електронний спектр квазі-2D кристалів

$$\varepsilon(k_x, k_y) \Rightarrow A(\omega, k_x, k_y)$$



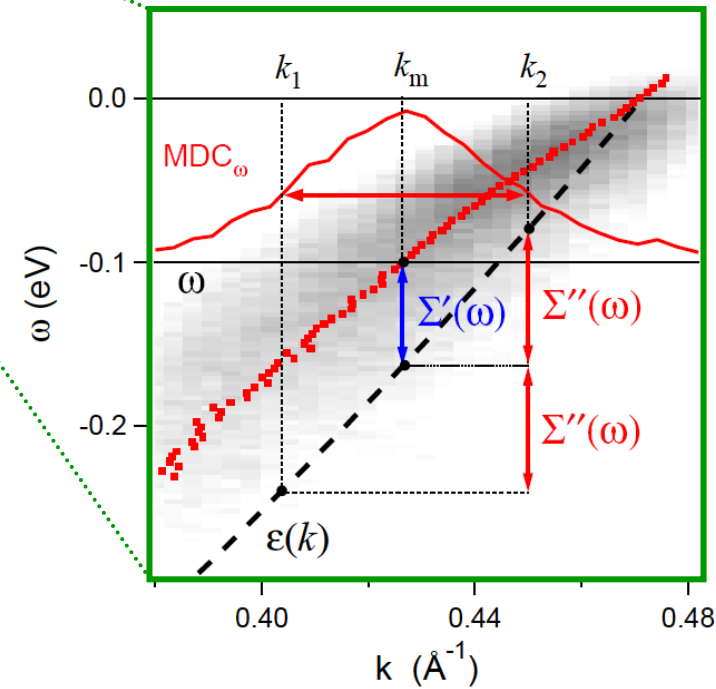
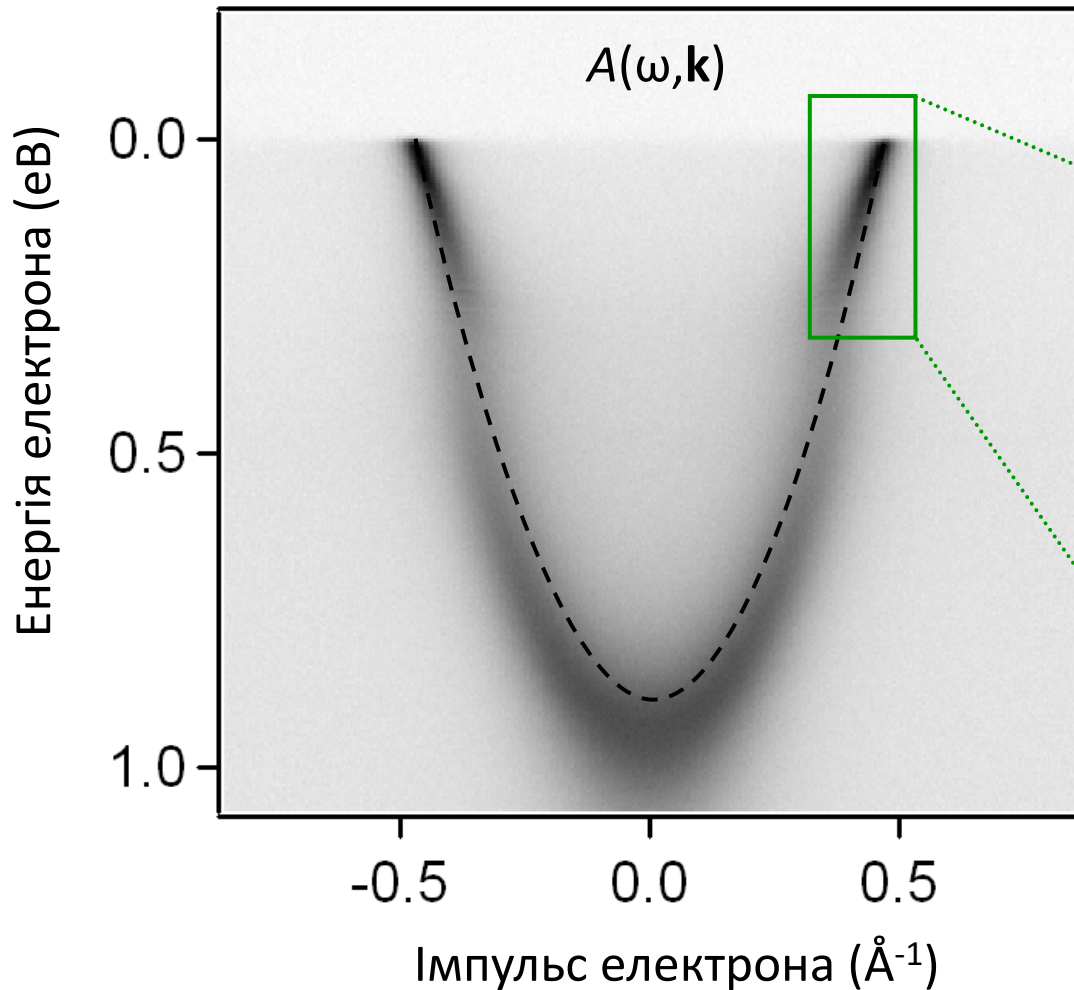
Структура електронного спектру

Спектральна функція

$$A(\omega, \mathbf{k}) = -\frac{1}{\pi} \frac{\Sigma''(\omega)}{(\omega - \varepsilon(\mathbf{k}) - \Sigma'(\omega))^2 + \Sigma''(\omega)^2}$$

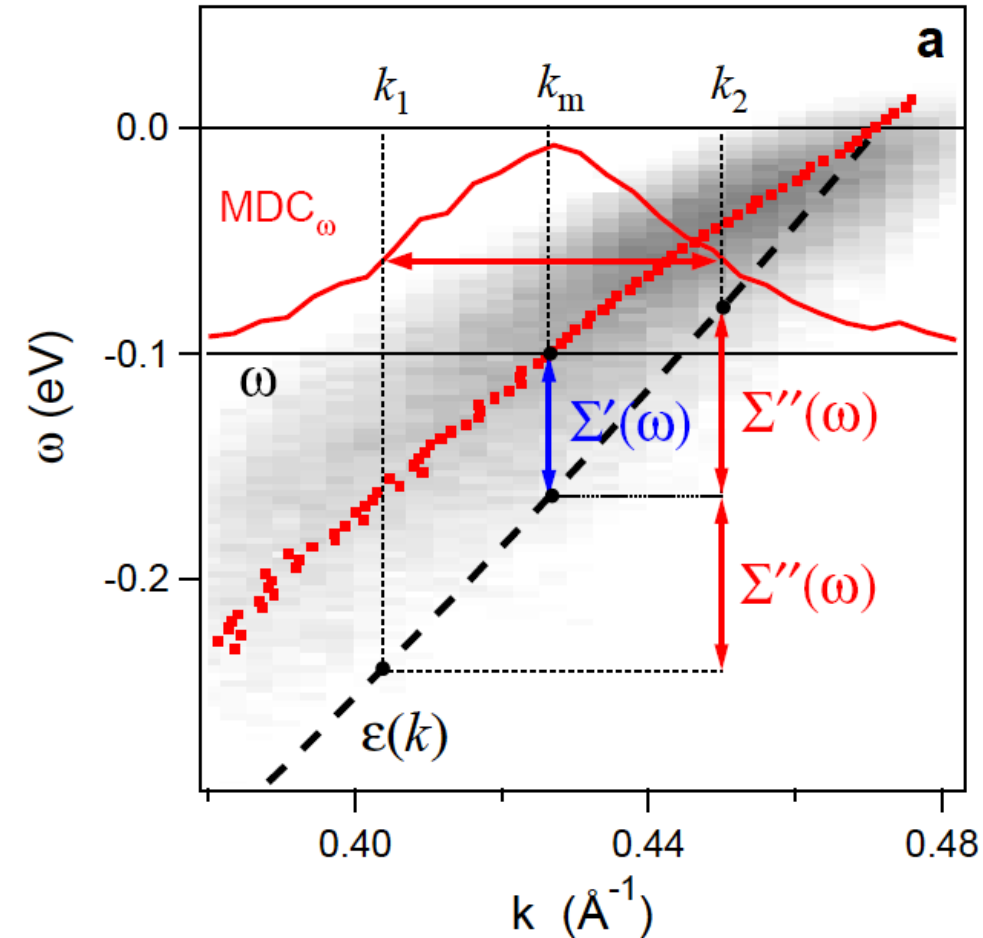
$\varepsilon(\mathbf{k})$ – "гола" електронна
зонна структура
 $\Sigma(\omega, \mathbf{k})$ – власна енергія

$$\Sigma'' = \hbar/\tau$$



Спектральна функція

$$A(\omega, \mathbf{k}) = -\frac{1}{\pi} \frac{\Sigma''(\omega)}{(\omega - \varepsilon(\mathbf{k}) - \Sigma'(\omega))^2 + \Sigma''(\omega)^2}$$

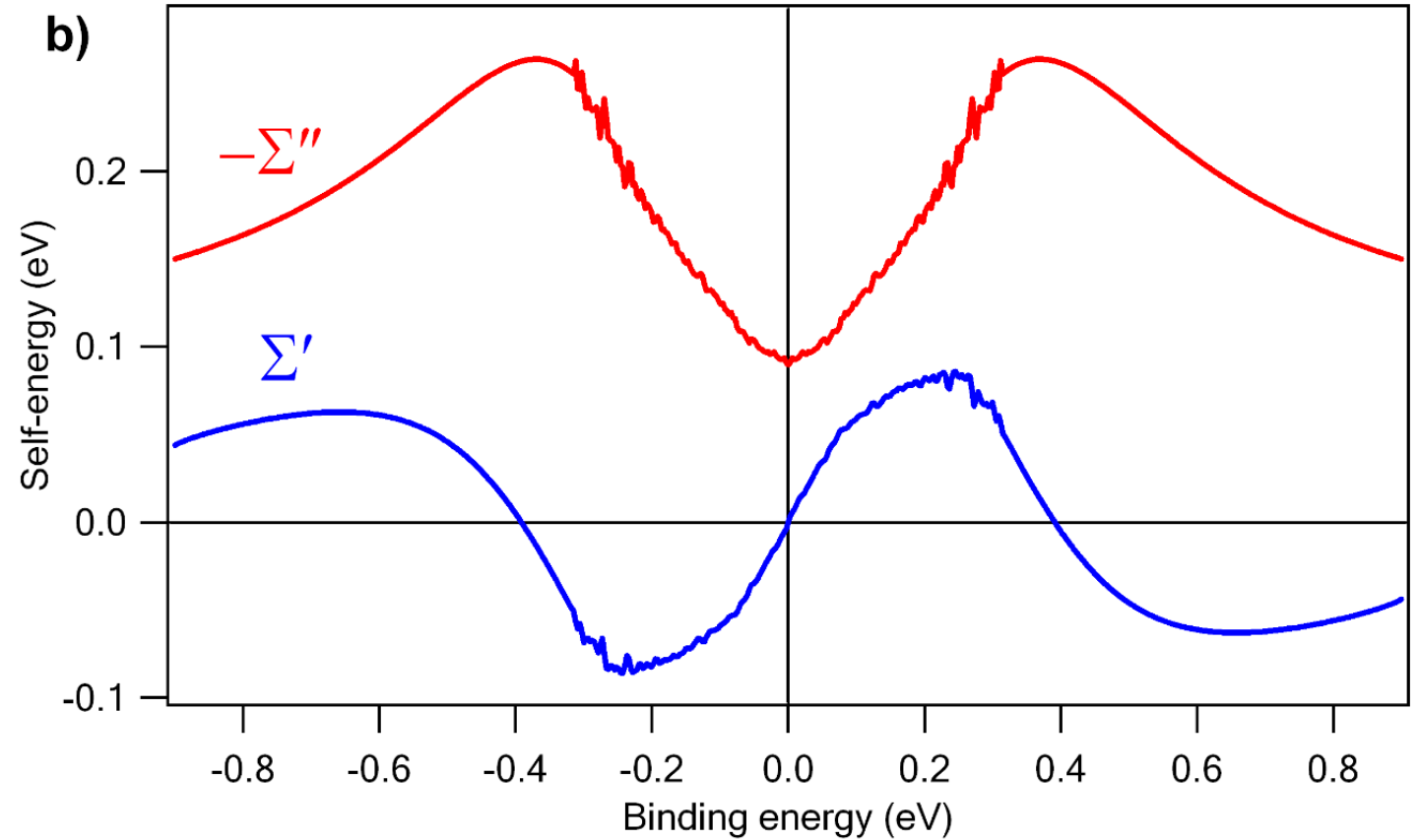
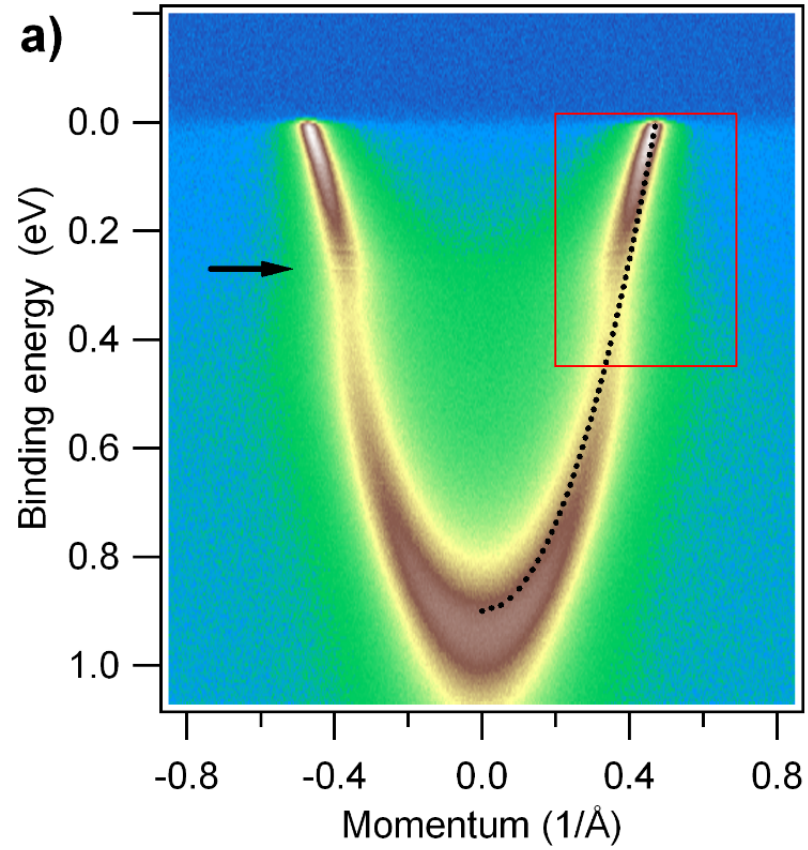


KK

$$\Sigma'(\omega) = \frac{1}{\pi} \text{PV} \int_{-\infty}^{\infty} \frac{\Sigma''(x)}{x - \omega} dx,$$

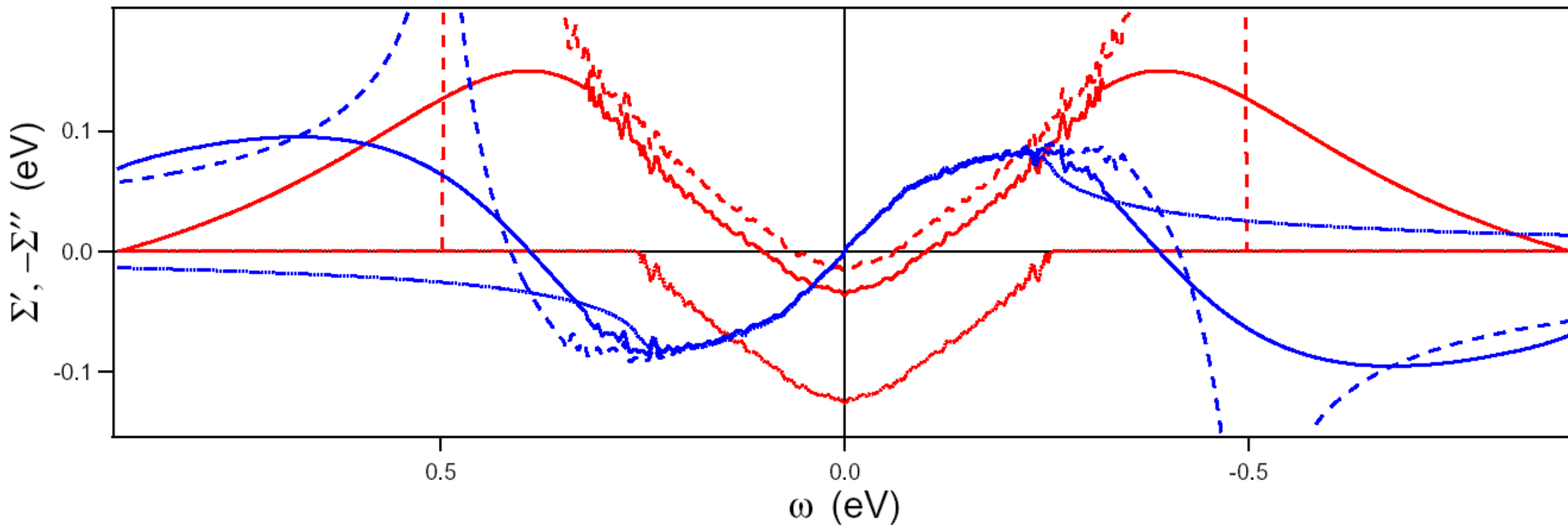
$$\Sigma''(\omega) = -\frac{1}{\pi} \text{PV} \int_{-\infty}^{\infty} \frac{\Sigma'(x)}{x - \omega} dx$$

Kramers-Kronig (KK) transformation



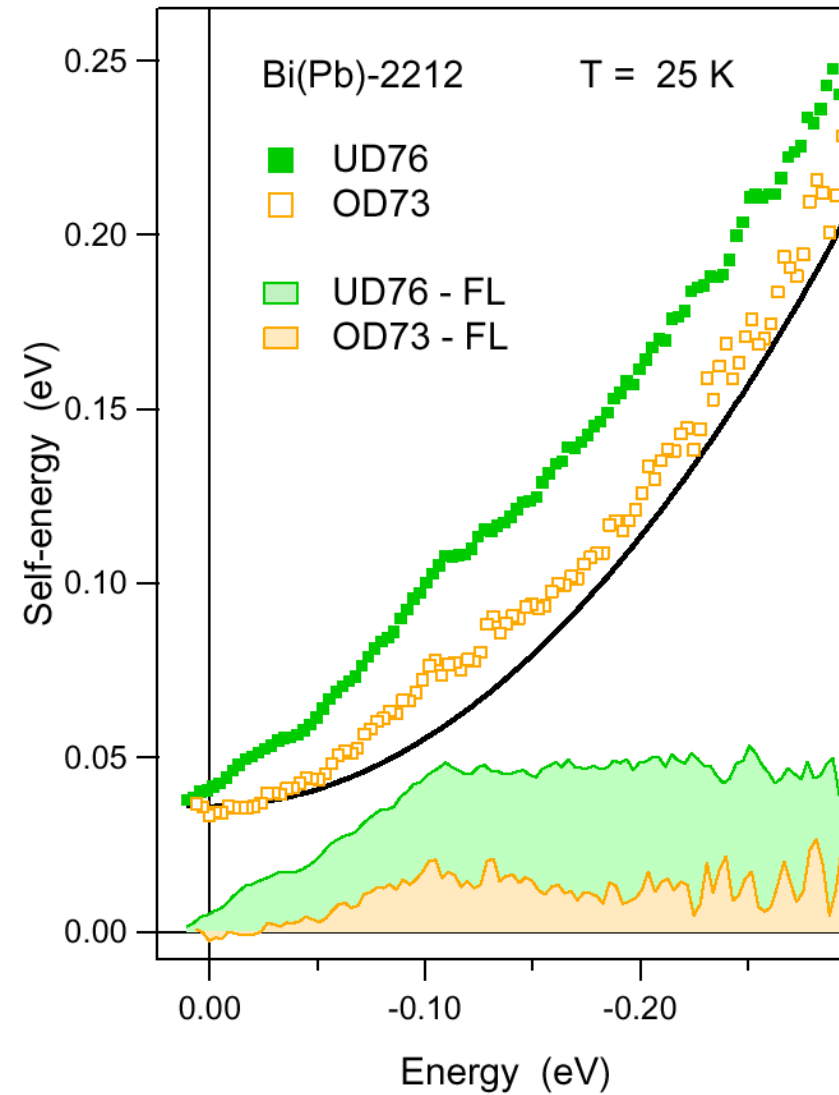
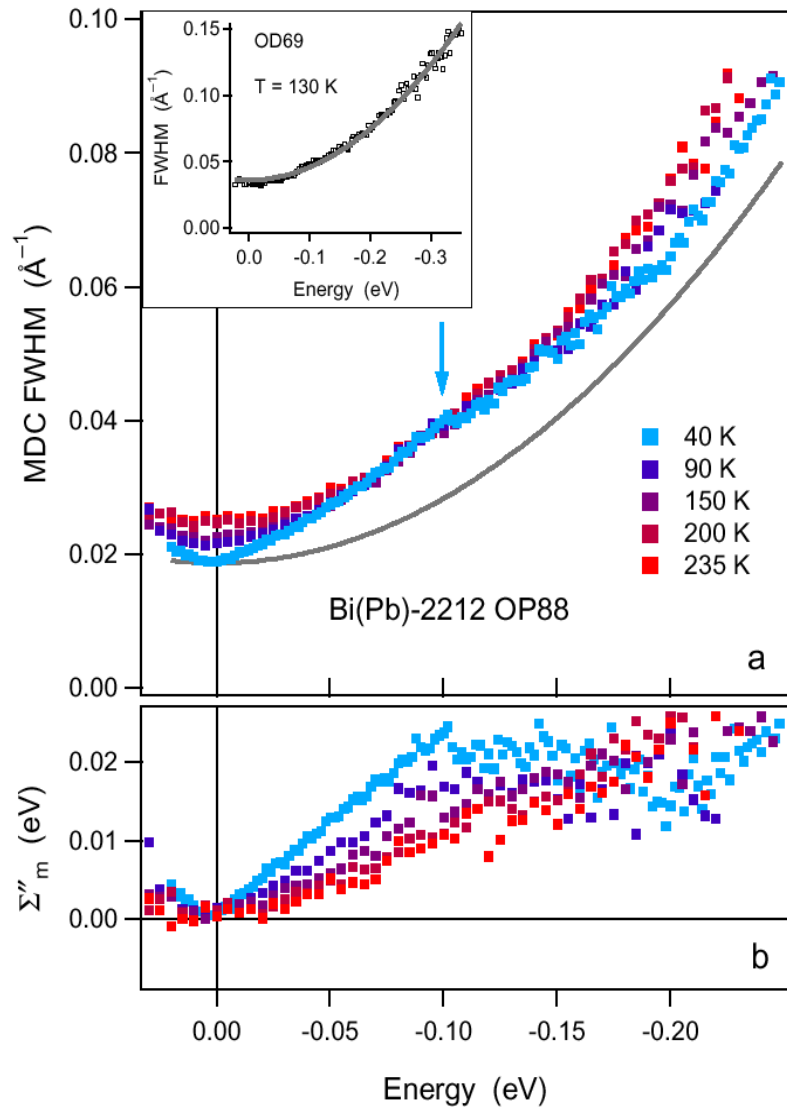
Kramers-Kronig transform

$$\Sigma'(\omega) = \text{KK} \Sigma''(\omega)$$



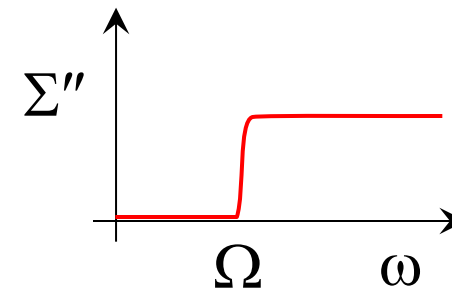
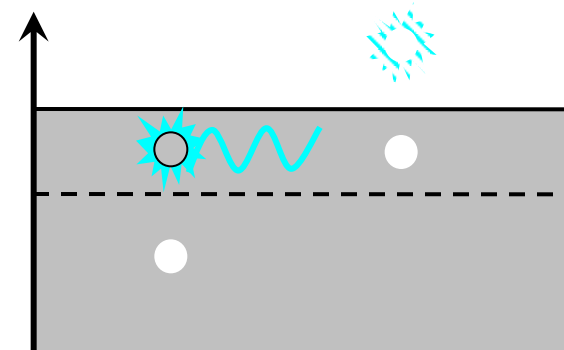
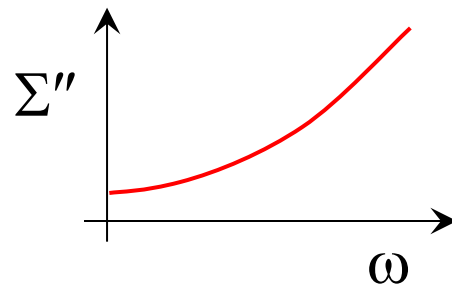
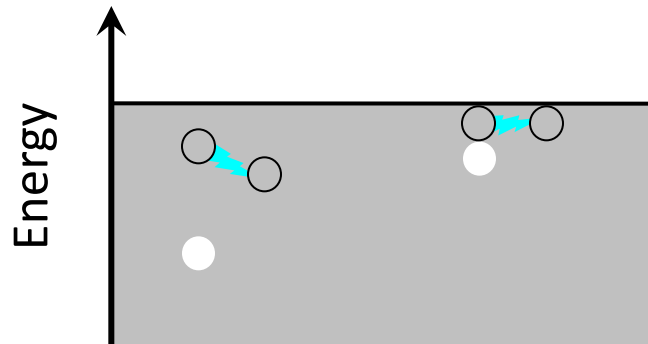
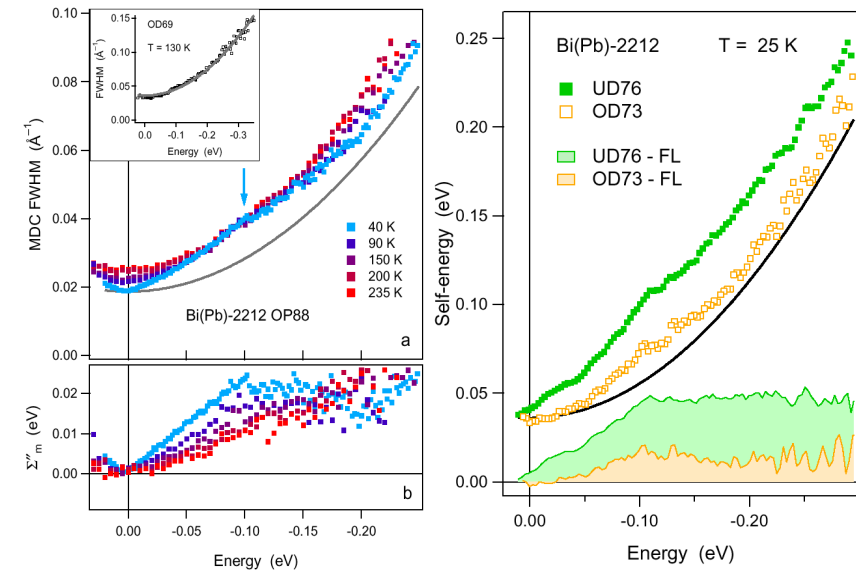
$$\Sigma''(\omega) = \begin{cases} \Sigma''_{width}(|\omega|) & \text{for } |\omega| < \omega_m, \\ \Sigma''_{mod}(\omega) & \text{for } |\omega| > \omega_m, \end{cases} \quad \Sigma''_{mod}(\omega) = -\frac{\alpha \omega^2 + C}{1 + \left|\frac{\omega}{\omega_c}\right|^n},$$

Scattering rate: T - and x -dependence



Scattering rate: Two channels

There are two channels:
1st electron-electron scattering and
2nd electron-boson scattering



Одно-частинкова спектральна функція = Im G

$$G_0(\omega, \mathbf{k}) = \frac{1}{\omega - \varepsilon(\mathbf{k}) + i\delta}$$

$$G(\omega, \mathbf{k}) = \frac{1}{\omega - \varepsilon(\mathbf{k}) - \Sigma(\omega)}$$

$$A(\omega, \mathbf{k}) = -\frac{1}{\pi} \text{Im } G(\omega, \mathbf{k})$$

$$A(\omega, \mathbf{k}) = -\frac{1}{\pi} \frac{\Sigma''(\omega)}{(\omega - \varepsilon(\mathbf{k}) - \Sigma'(\omega))^2 + \Sigma''(\omega)^2}$$

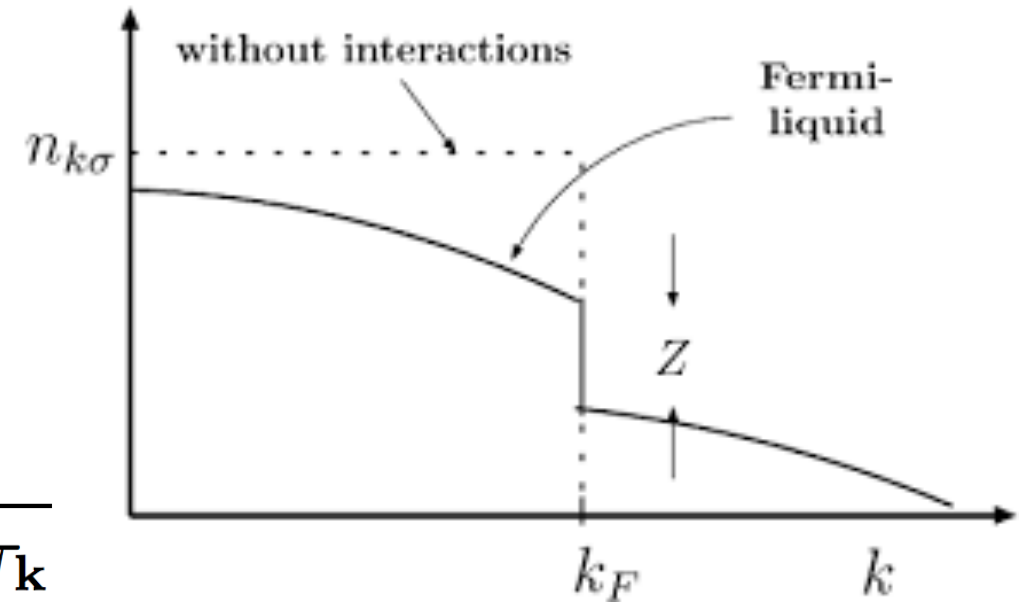
$$(G(\mathbf{k}, \omega))^{-1} = (G^0(\mathbf{k}, \omega))^{-1} - \Sigma(\mathbf{k}, \omega)$$

Landau Fermi liquid

$$G(\mathbf{k}, \omega) = G_{coh}(\mathbf{k}, \omega) + G_{incoh}(\mathbf{k}, \omega)$$

$$G_{coh}(\mathbf{k}, \omega) = \frac{Z_{\mathbf{k}}}{\omega - \tilde{\epsilon}_{\mathbf{k}} + i/\tau_{\mathbf{k}}}$$

$$Z_{\mathbf{k}} = \left(1 - \frac{\partial \Sigma}{\partial \omega} \right)_{\omega=0, k=k_F}^{-1} \quad \frac{\Sigma''(\omega)}{\omega} \rightarrow 0$$



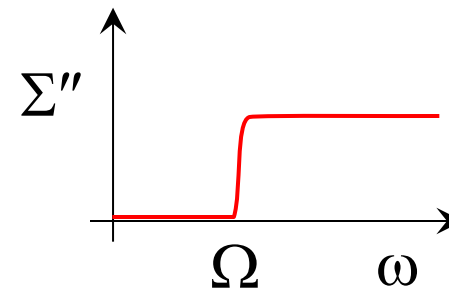
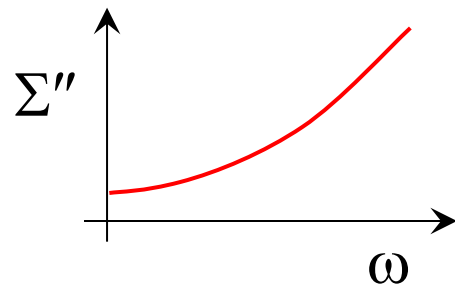
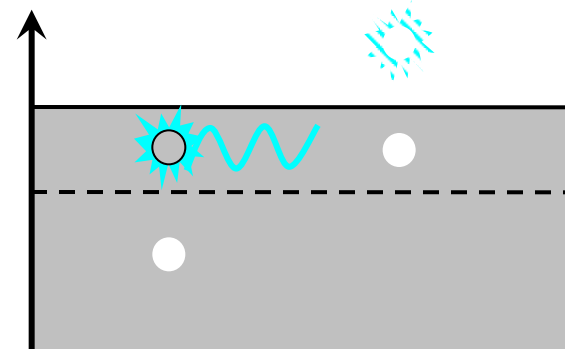
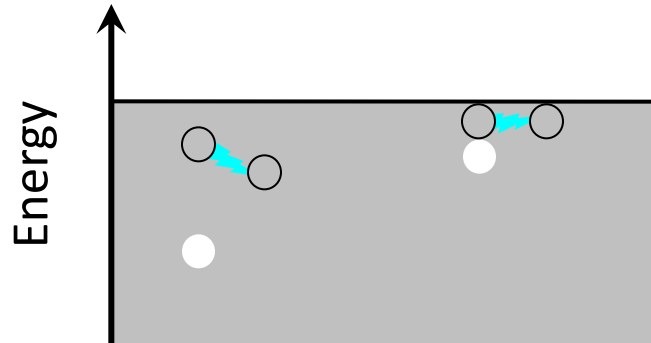
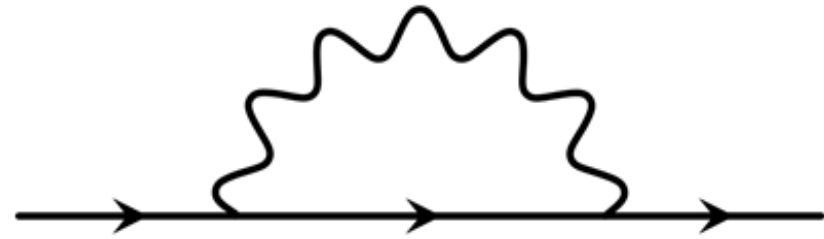
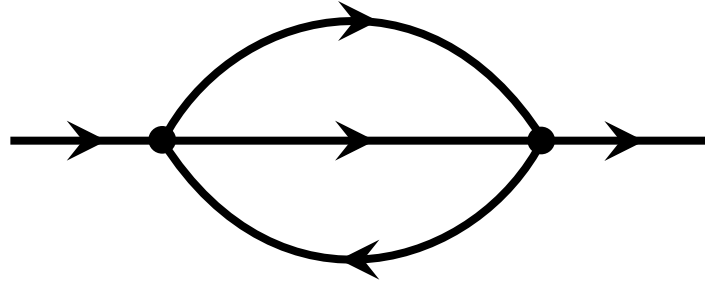
А.А.АБРИКОСОВ, А.П.ГОРЬКОВ
И.Е.ДЗЯЛОВИНСКИЙ

МЕТОДЫ
КВАНТОВОЙ ТЕОРИИ ПОЛЯ
В СТАТИСТИЧЕСКОЙ
ФИЗИКЕ

<https://arxiv.org/pdf/cond-mat/0103393.pdf>

Landau, L. D. The theory of a Fermi liquid. *J. Exp. Theor. Phys.* **30**, 1058–1064 (1956).

Feynman's diagrams



Dirac Delta Function and Green's Function

$$\delta(x - x') = \lim_{n \rightarrow \infty} f_n(x) = \begin{cases} 0, & x \neq x' \\ \infty, & x = x' \end{cases}, \quad \int_{-\infty}^{\infty} dx f_n(x) = 1, \quad \forall n.$$

$$g \mapsto \int_{-\infty}^{\infty} dx \delta(x - x') g(x) = g(x')$$

$$\mathcal{L}G(x, x') = \delta(x - x')$$

$$\mathcal{L}u(x) = f(x) \qquad u(x) = \int dx' G(x, x') f(x')$$

Quantum Green's functions

$$\left[i\hbar \frac{\partial}{\partial t} + \frac{\hbar^2}{2m} \nabla^2 \right] \Psi(\mathbf{r}, t) = V(\mathbf{r}, t) \Psi(\mathbf{r}, t)$$

$$\left[i\hbar \frac{\partial}{\partial t} + \frac{\hbar^2}{2m} \nabla^2 \right] G(\mathbf{r}, t; \mathbf{r}', t') = \delta(\mathbf{r} - \mathbf{r}') \delta(t - t')$$

$$\Psi(\mathbf{r}, t) = \int G(\mathbf{r}, t; \mathbf{r}', t') \Psi(\mathbf{r}', t') d^3 r'$$

This is why the Green's function is known as the *propagator*

$$G(\mathbf{r}, t; \mathbf{r}', t') = \langle \mathbf{r} | e^{-\frac{i}{\hbar} H(t-t')} | \mathbf{r}' \rangle = \langle \mathbf{r}, t | \mathbf{r}', t' \rangle$$

Green's function is the *probability amplitude* of finding the particle in a state $\langle \mathbf{r}, t |$ given that it started at $| \mathbf{r}', t' \rangle$

Electron Green's function: Spectral representation

$$G_{ij}^r(t, t') = -i\theta(t - t') \left\langle \left\{ c_i(t), c_j^\dagger(t') \right\} \right\rangle$$

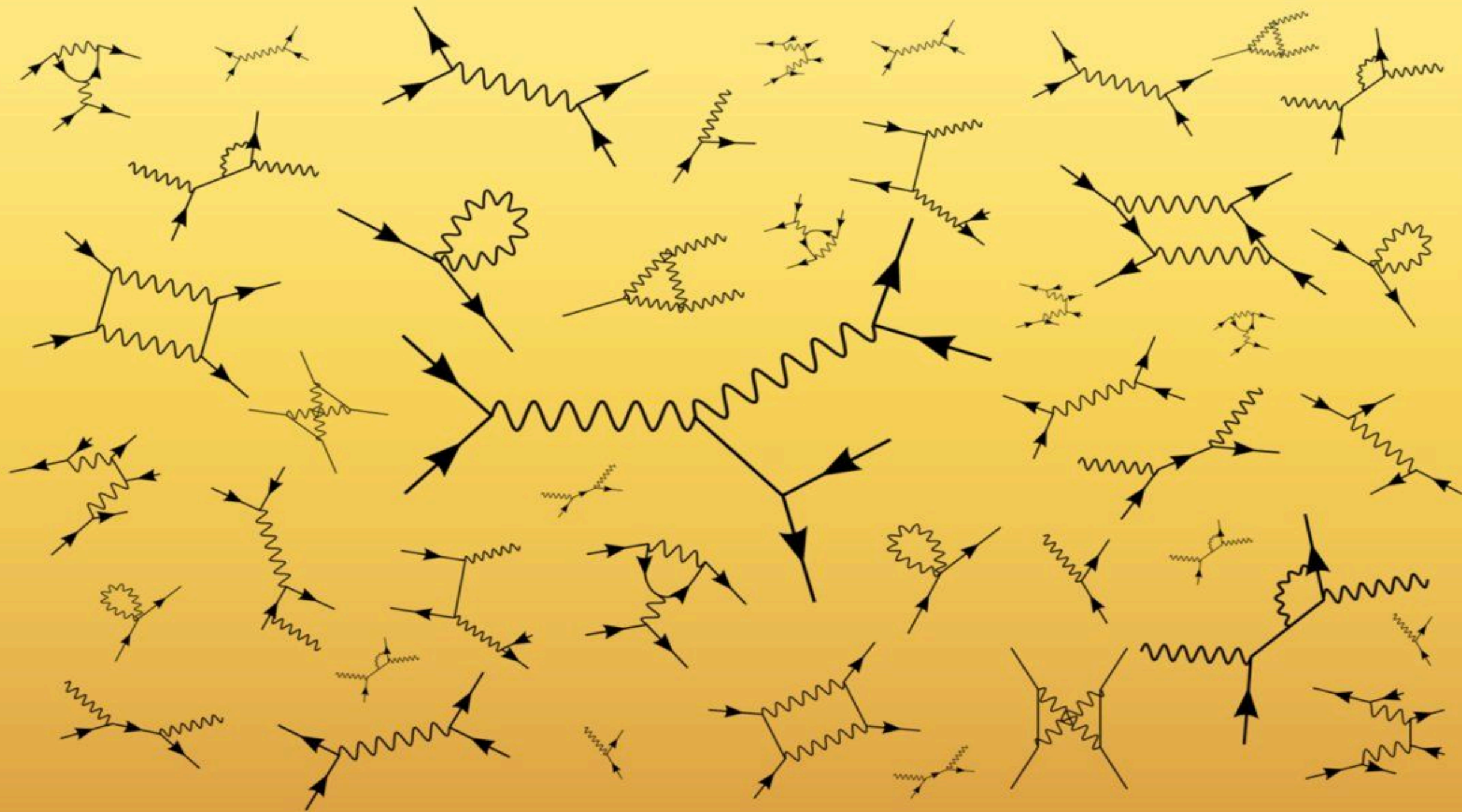
retarded Green's function is non-zero only for $t > t'$

$$G_{nn'}^r(t - t') = -i\theta(t - t') e^{-i\varepsilon_n(t-t')} \delta_{nn'} \quad \langle \{c_n, c_{n'}^\dagger\} \rangle = \delta_{nn'}$$

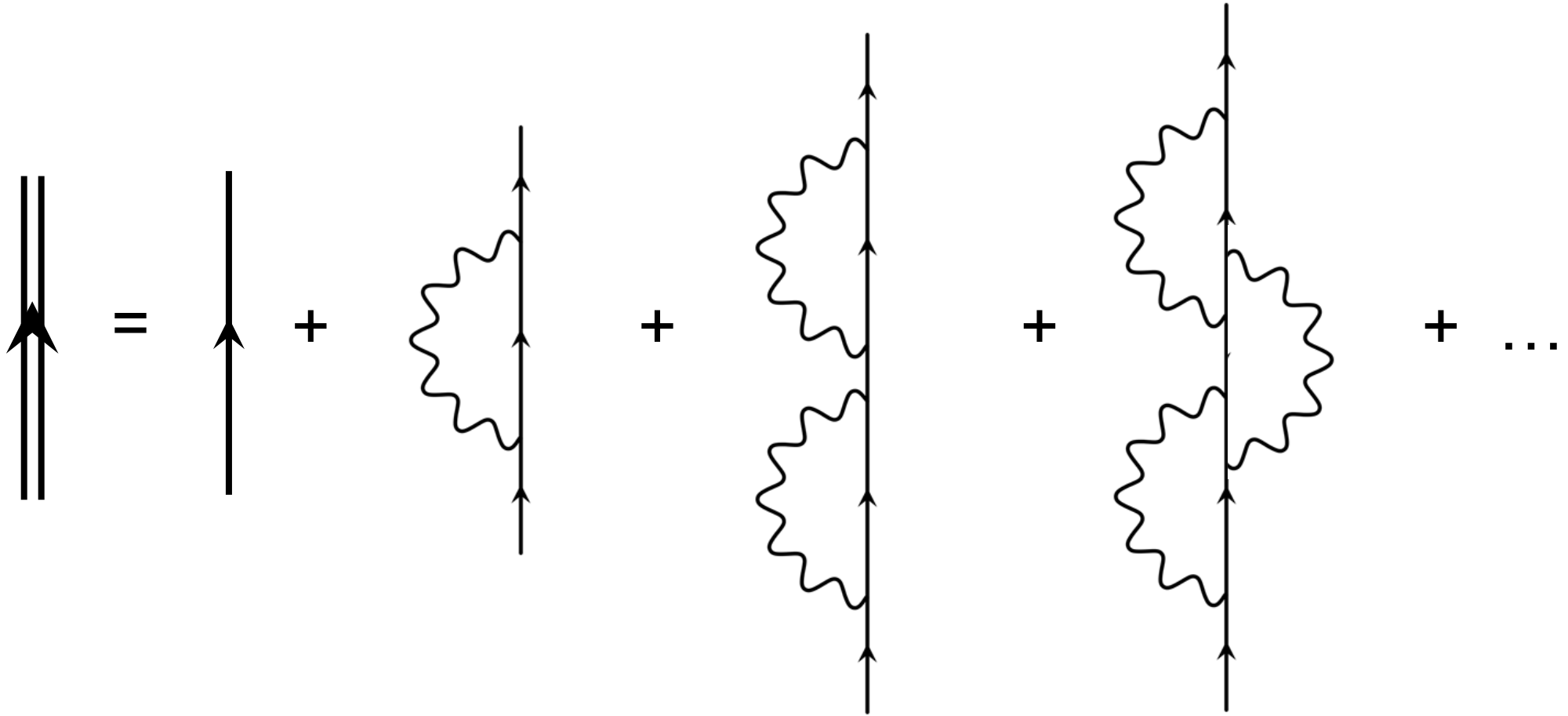
$$\theta(t - t') = -\frac{1}{2\pi i} \int_{-\infty}^{\infty} d\omega \frac{e^{-i\omega(t-t')}}{\omega + i\eta}$$

$$G_{nn}^r(t - t') = \frac{1}{2\pi} \int_{-\infty}^{\infty} d\omega \frac{e^{-i\omega(t-t')}}{\omega - \varepsilon_n + i\eta}$$

$$G_{nn}^r(\omega) = \frac{1}{\omega - \varepsilon_n + i\eta}$$



Electron Green's function: Feynman's diagrams



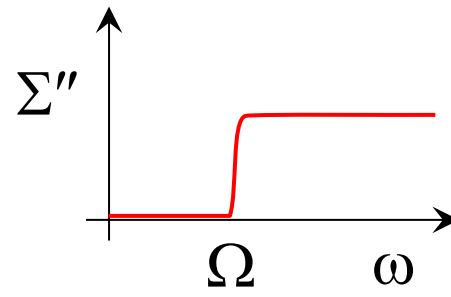
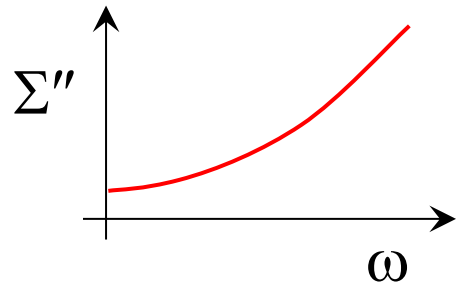
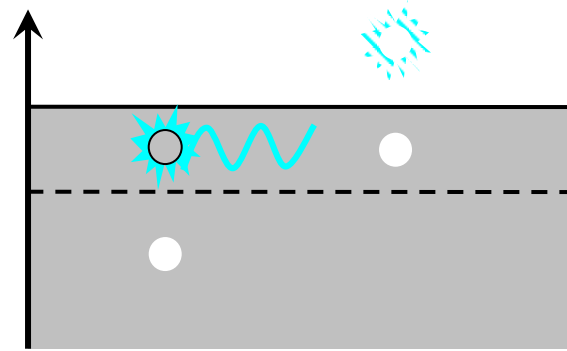
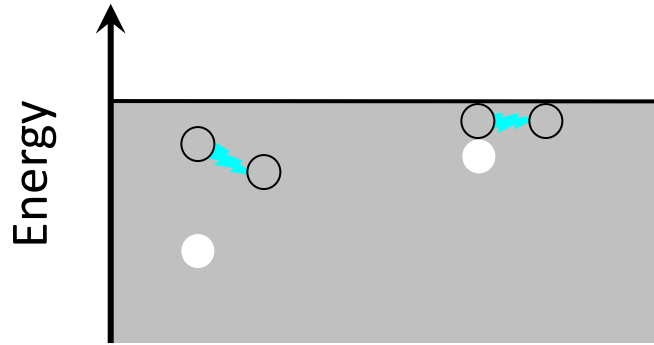
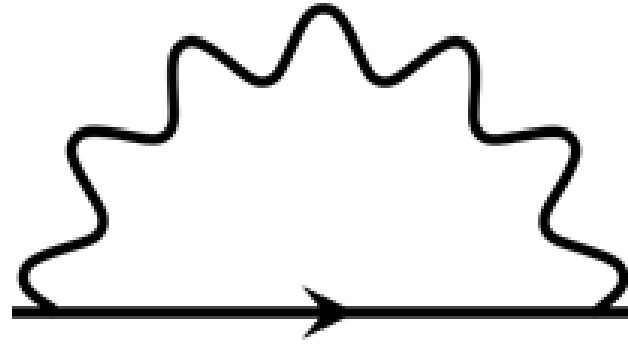
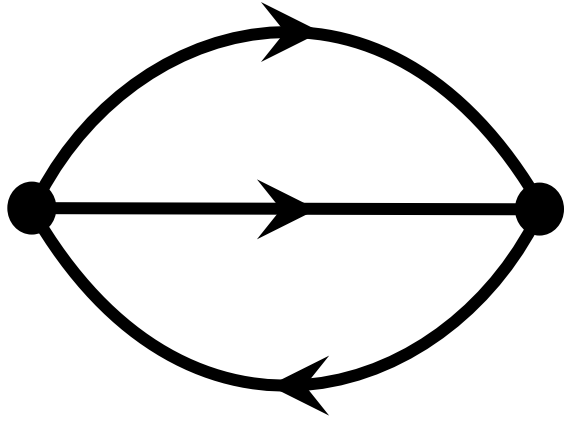
$$G = G_0 + G_0 \Sigma G_0 + G_0 (\Sigma G_0)^2 + G_0 (\Sigma G_0)^3 + \dots$$

Electron Green's function: Dyson equation

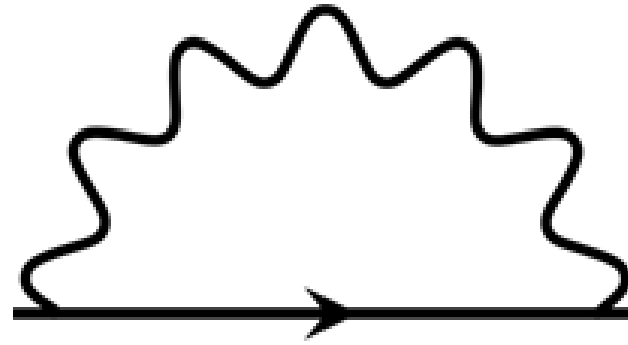
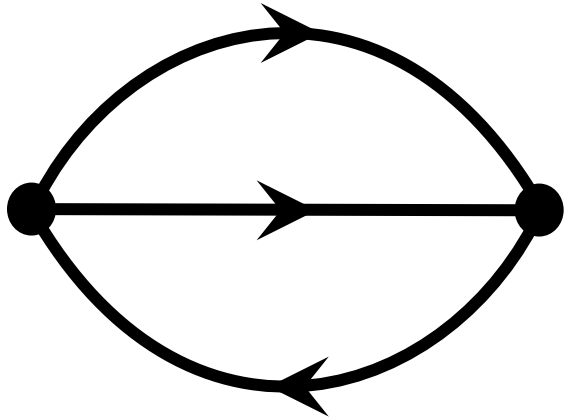
$$\begin{aligned} G &= G_0 + G_0 \Sigma G_0 + G_0 (\Sigma G_0)^2 + G_0 (\Sigma G_0)^3 + \dots = \\ &= G_0 + G_0 \Sigma (G_0 + G_0 \Sigma G_0 + G_0 (\Sigma G_0)^2 + \dots) = G_0 + G_0 \Sigma G \end{aligned}$$

$$G = \frac{G_0}{1 - G_0 \Sigma} = \frac{1}{G_0^{-1} - \Sigma} = \frac{1}{\omega - \varepsilon - \Sigma}$$

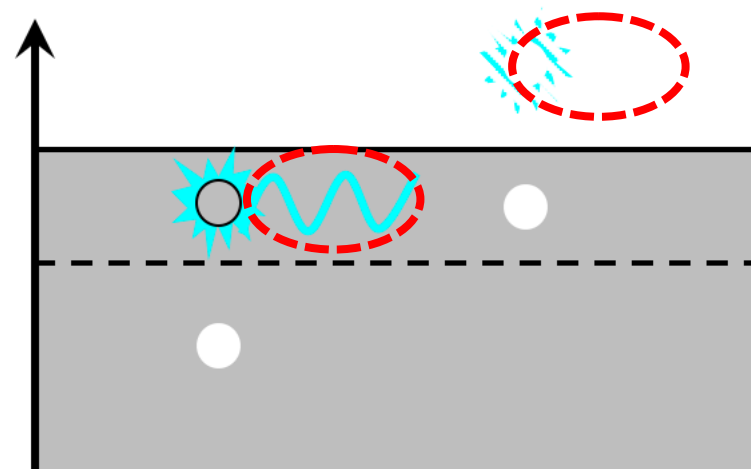
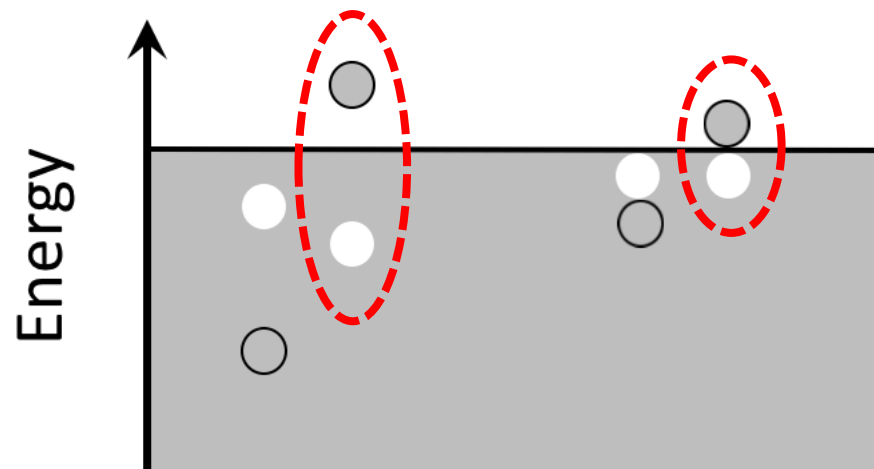
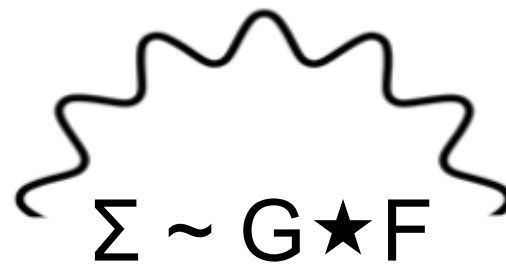
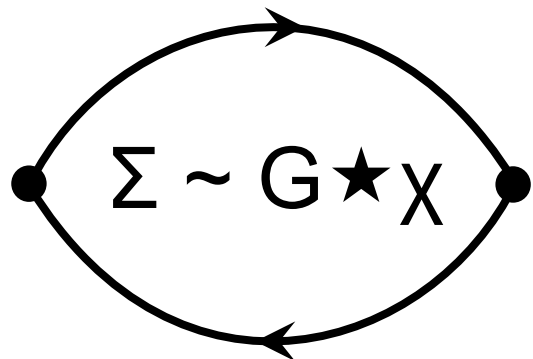
$$G^{-1} = G_0^{-1} - \Sigma$$



1 particle & 2 particle spectral functions



$$\Sigma \sim (G \star X)_{\mathbf{k}, \omega}$$



$$\chi \sim G \star G$$

$$\alpha^2 F$$

Двохчастинкова функція – функція Ліндгарда

$$\chi(\Omega, \mathbf{Q})$$

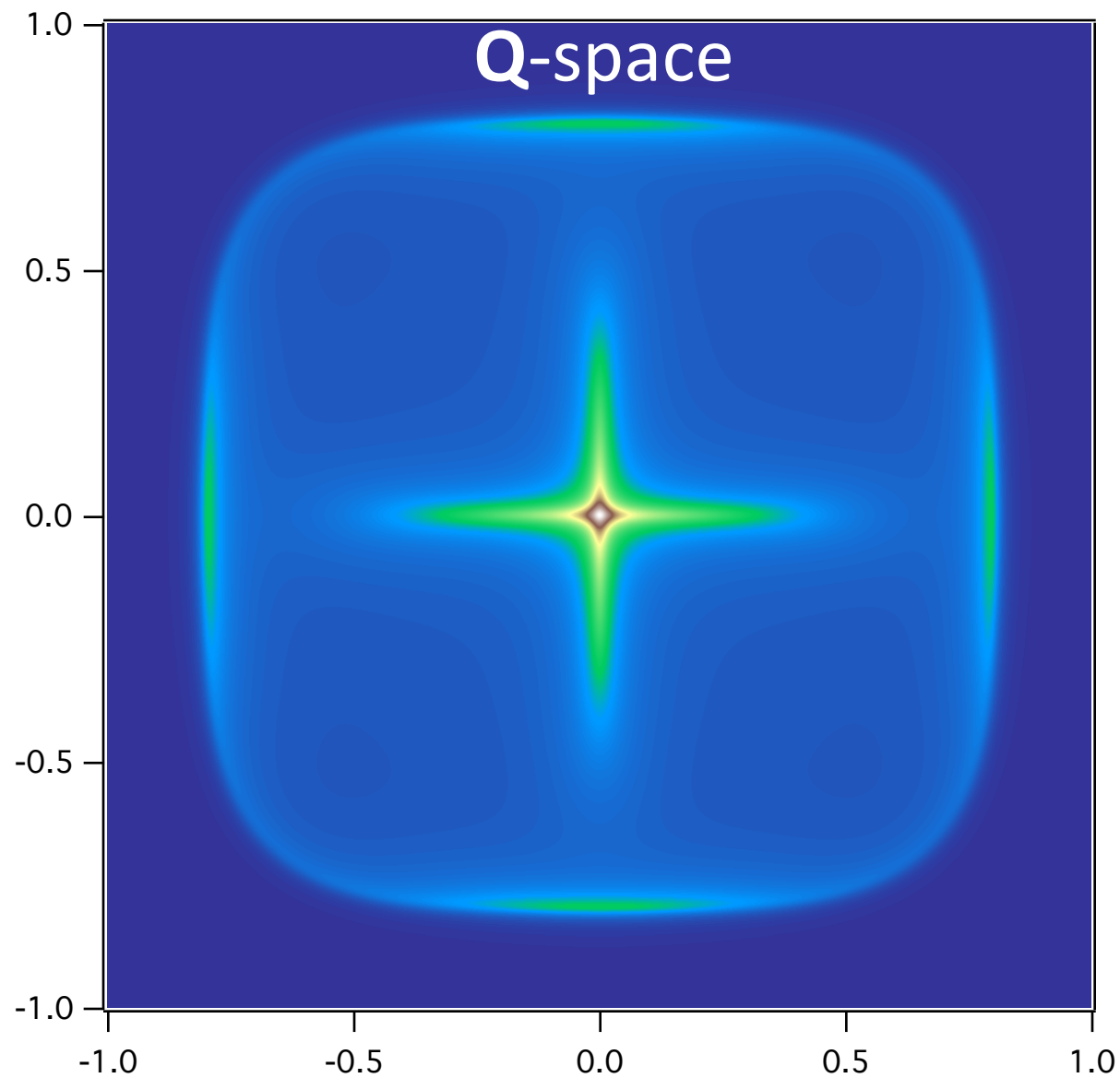
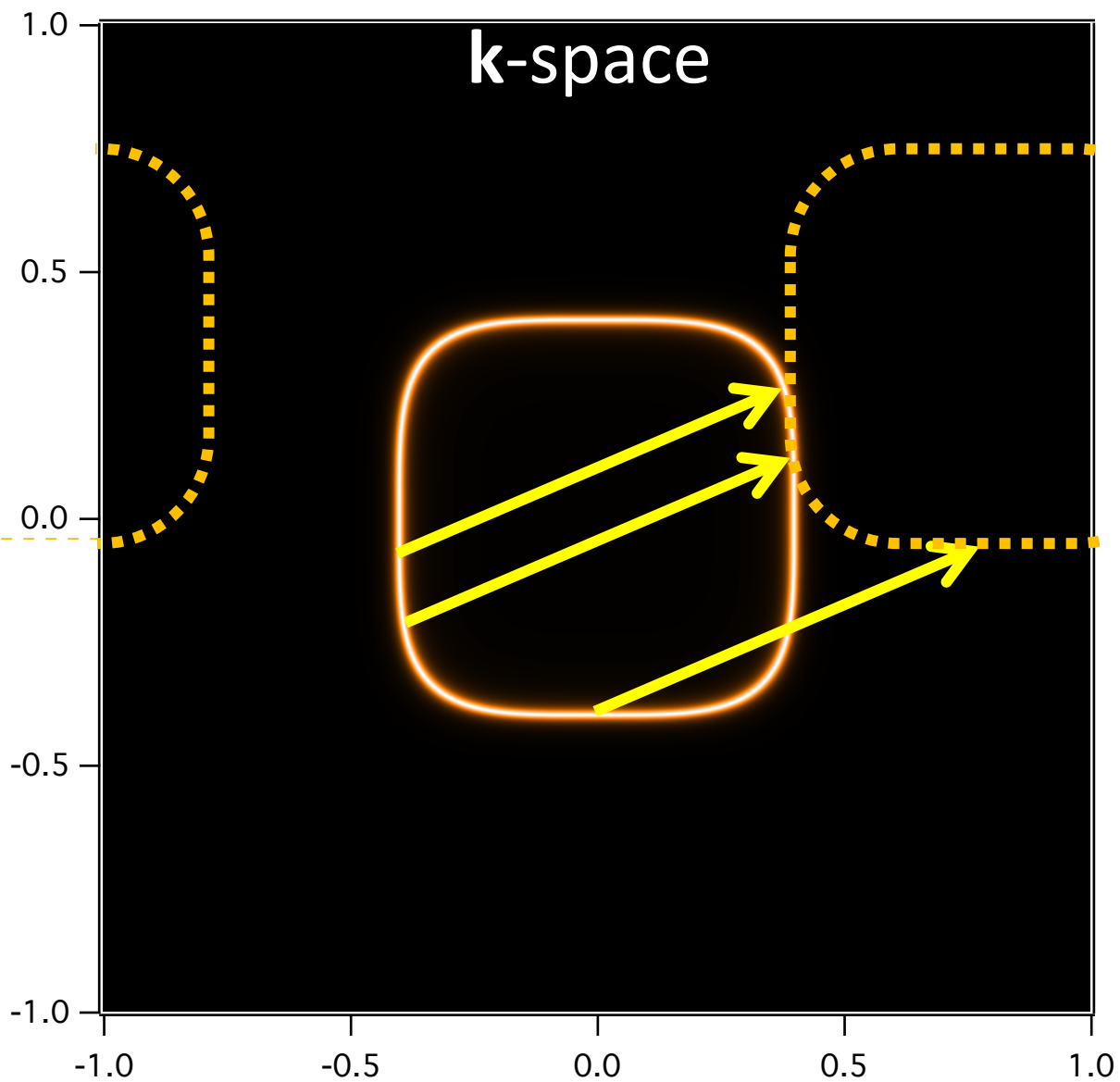
$$\chi_0(\mathbf{Q}, i\Omega_n) = \frac{1}{\pi^2} \int \sum_m G(\mathbf{k}, i\omega_m) G(\mathbf{k} + \mathbf{Q}, i\omega_m + i\Omega_n) d\mathbf{k}$$

\mathbf{Q}, Ω - space \mathbf{k}, ω - space

$$\chi \sim G \star G$$

Numerical (auto)correlation

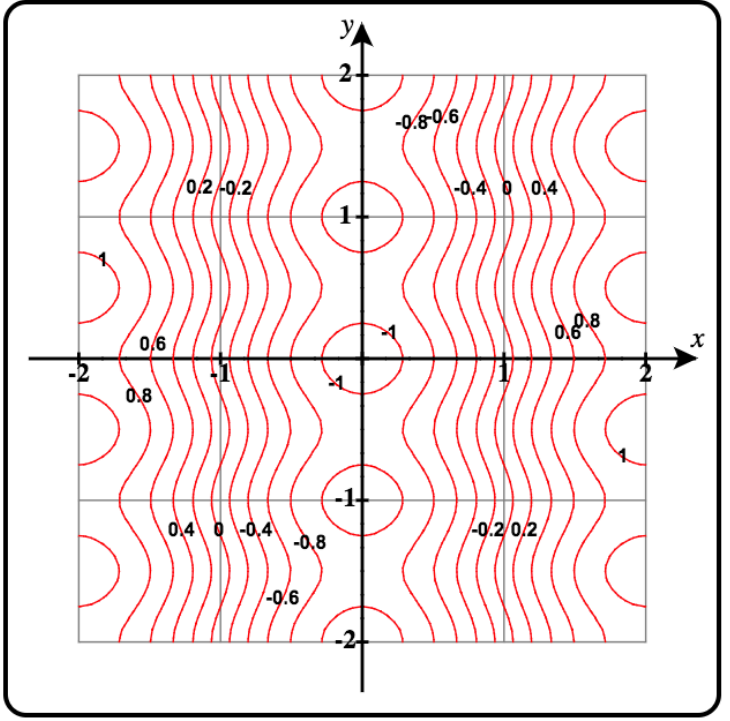
$$destWaveOut[p] = \sum_{m=0}^{N-1} srcWave[m] \cdot destWaveIn[p+m]$$





F

Graph 3D Mode



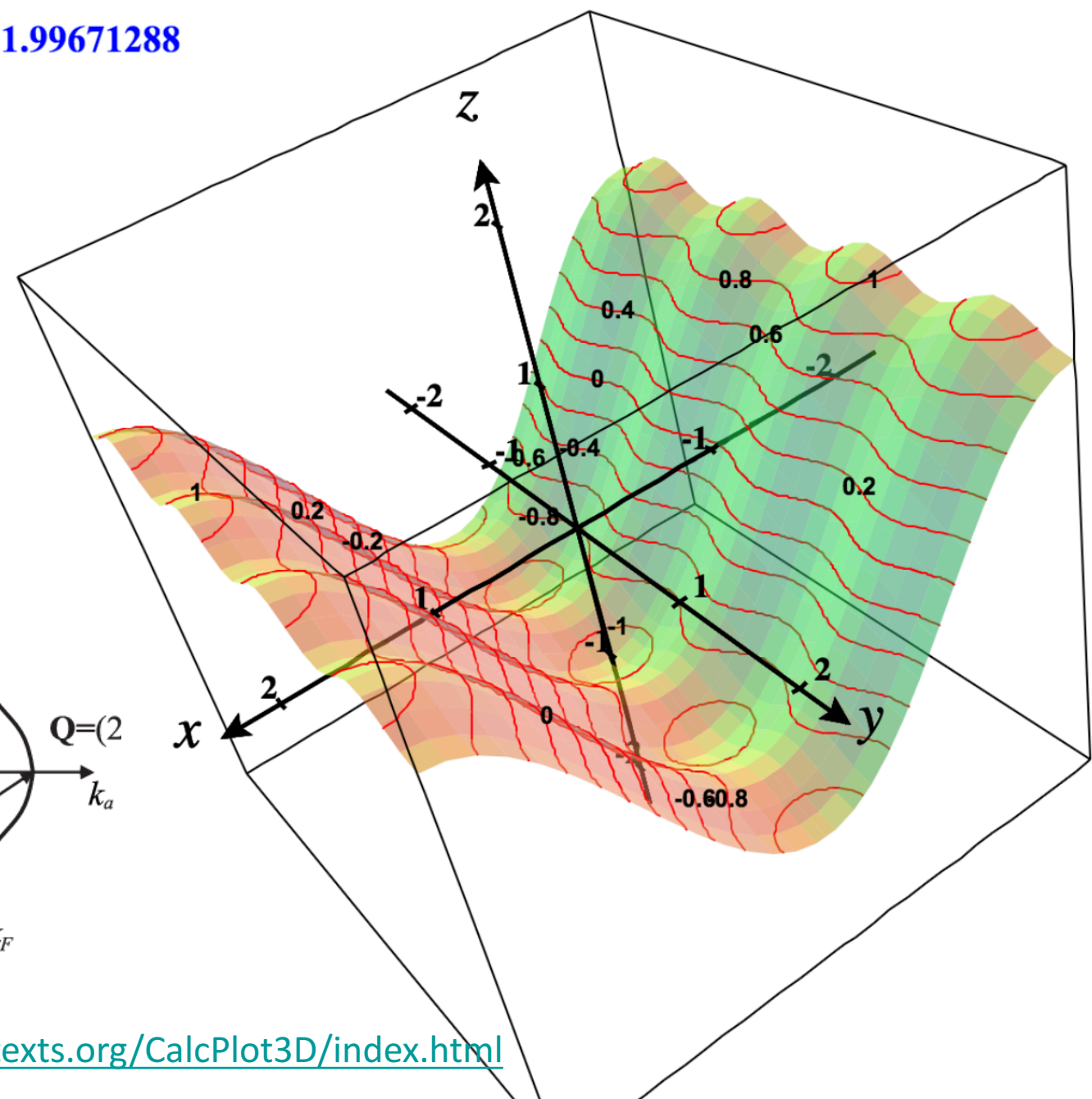
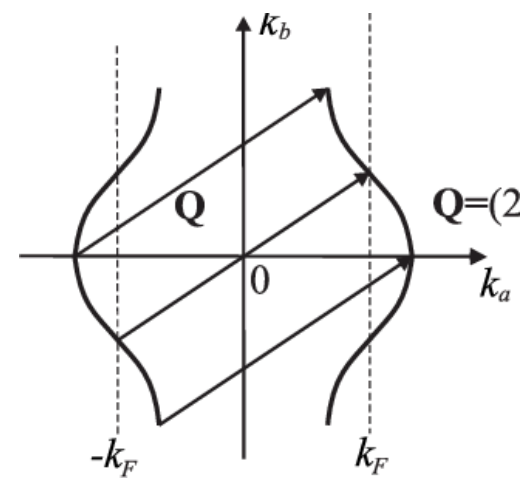
x = 1.963
y = 1.963

Add to graph: Select...

z = -cos(pi*x/2)-0.1*cos(4*pi*y/2)

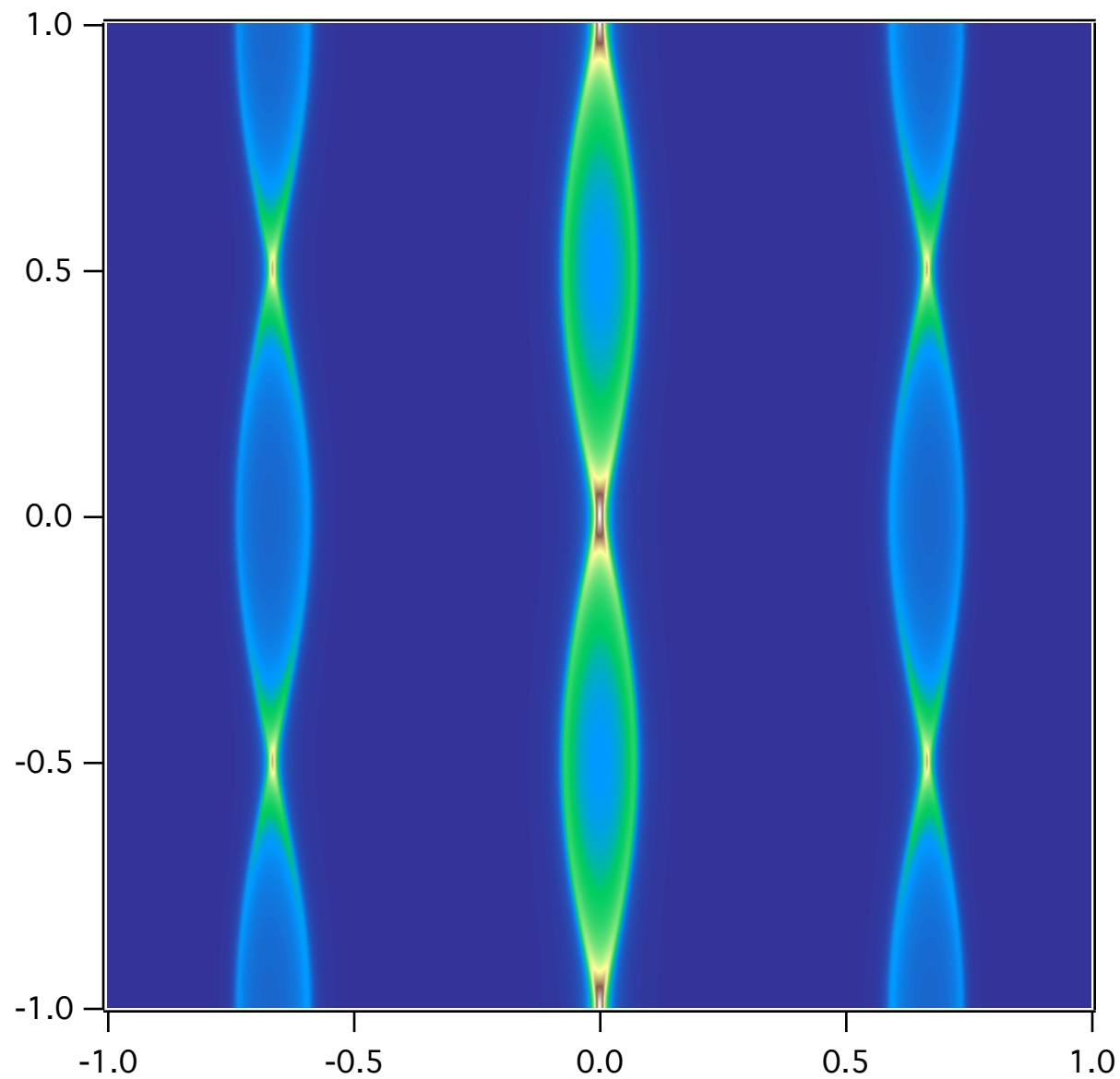
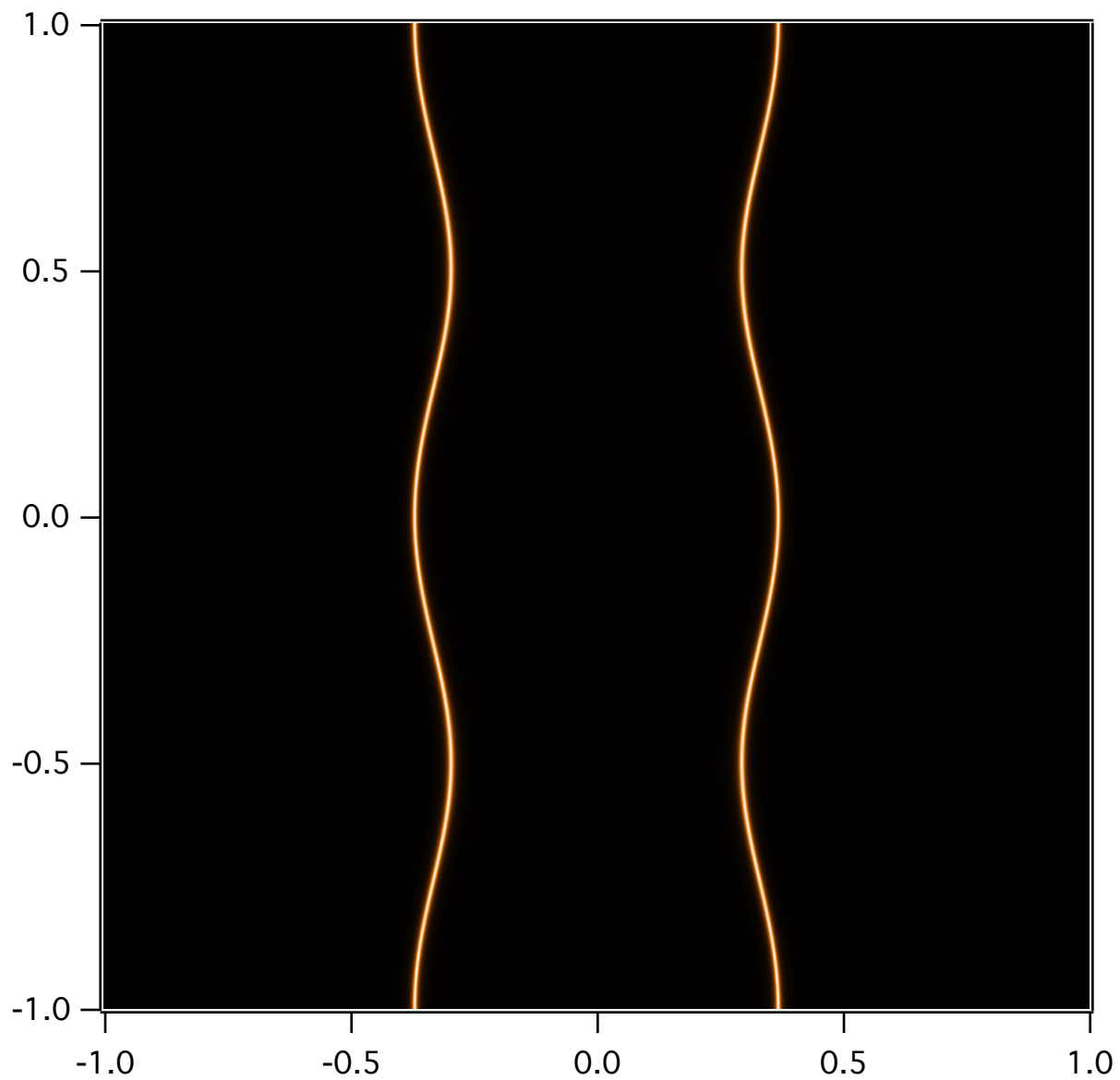
Number of Gridlines 30

$f(1.9635, 1.9635) = 1.99671288$



<https://c3d.libretexts.org/CalcPlot3D/index.html>

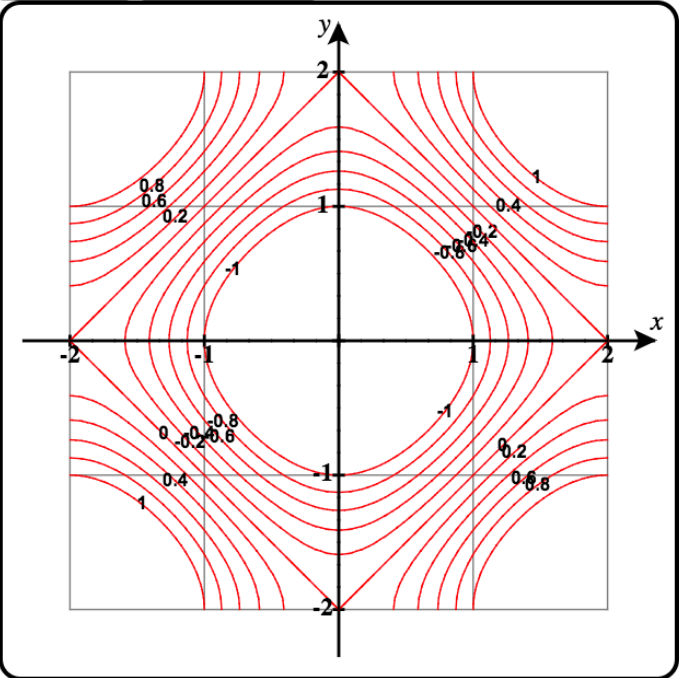
Numerical autocorrelation





F

Graph 3D Mode



x = 1.963

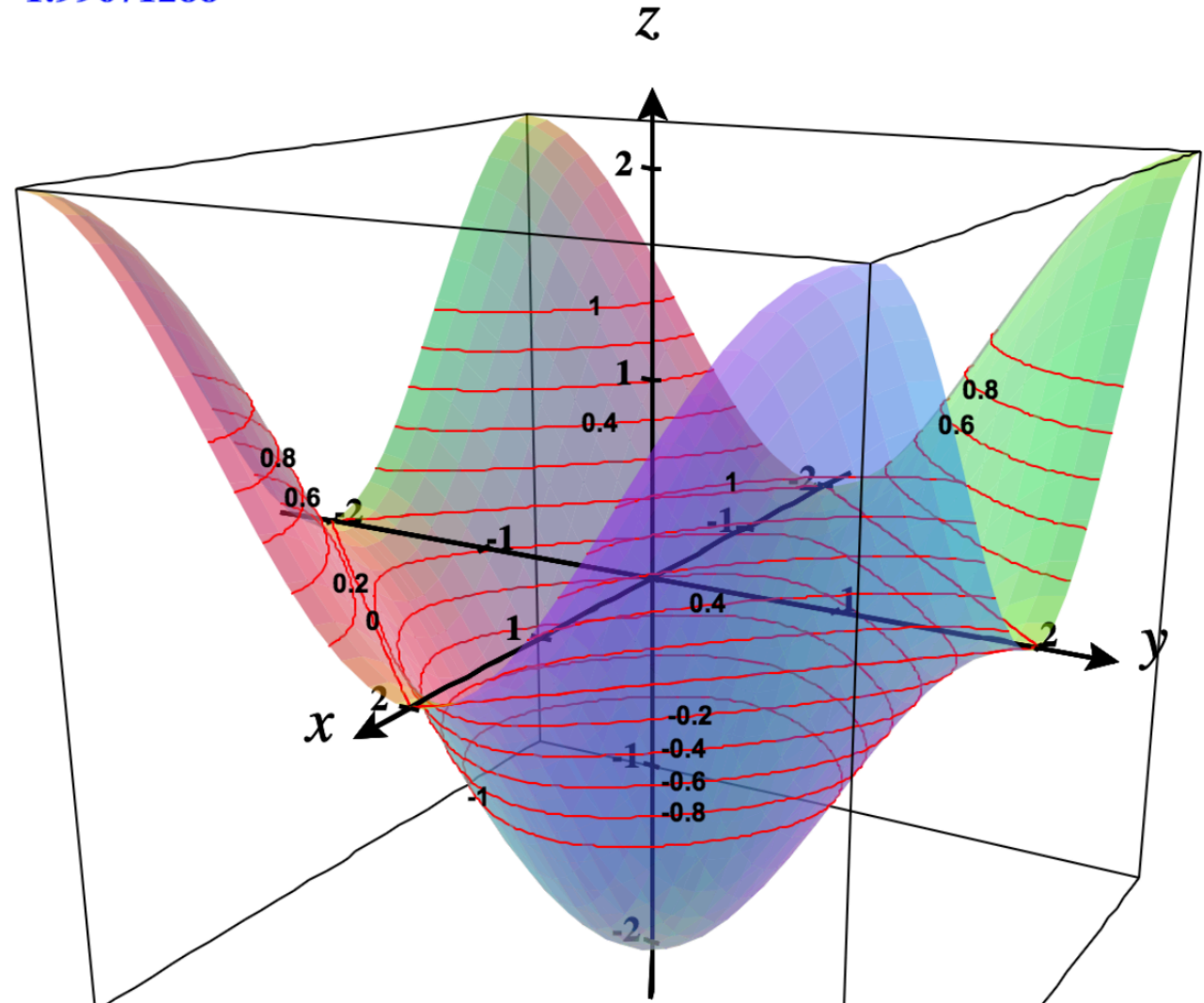
y = 1.963

Add to graph: Select...

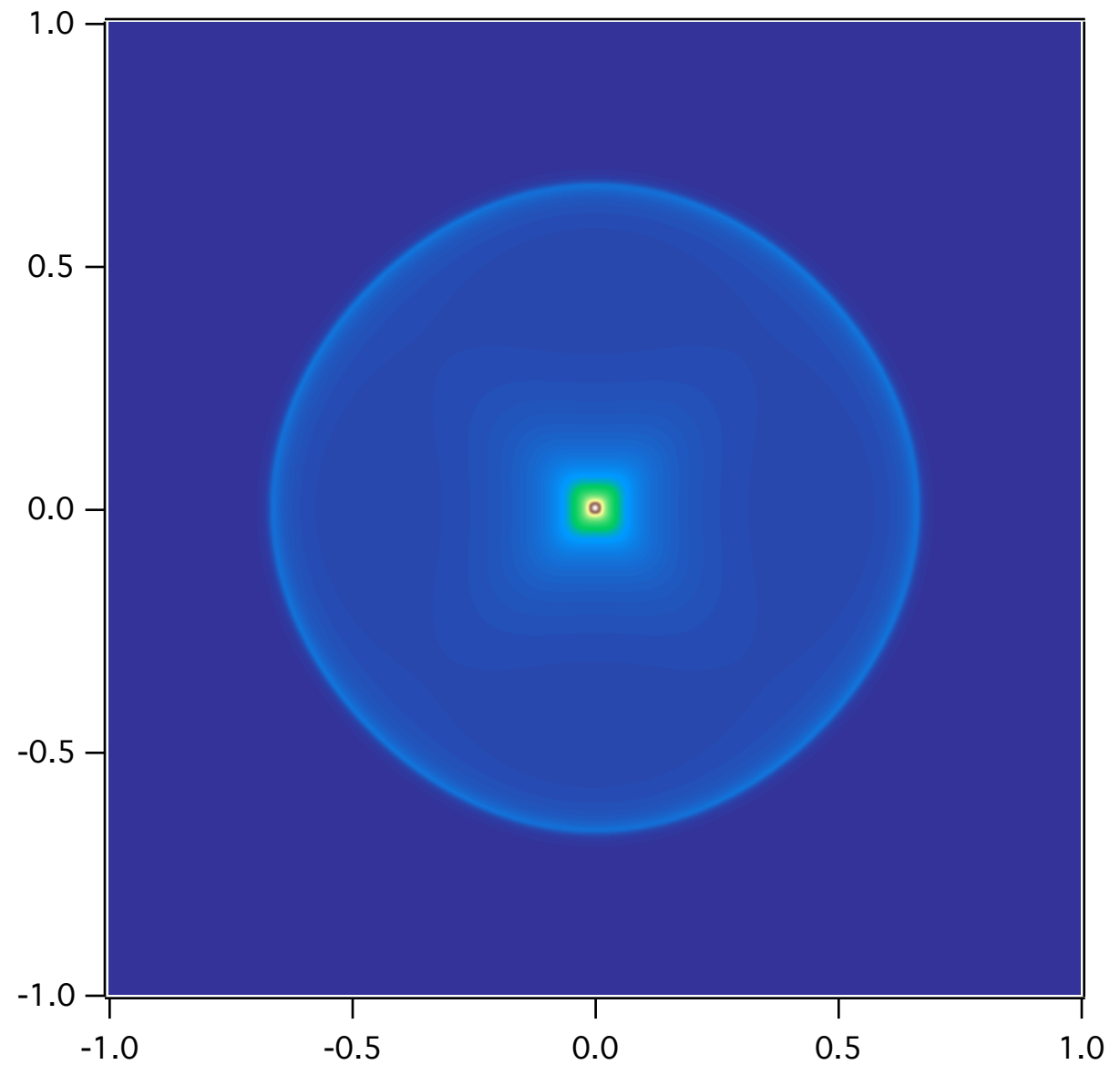
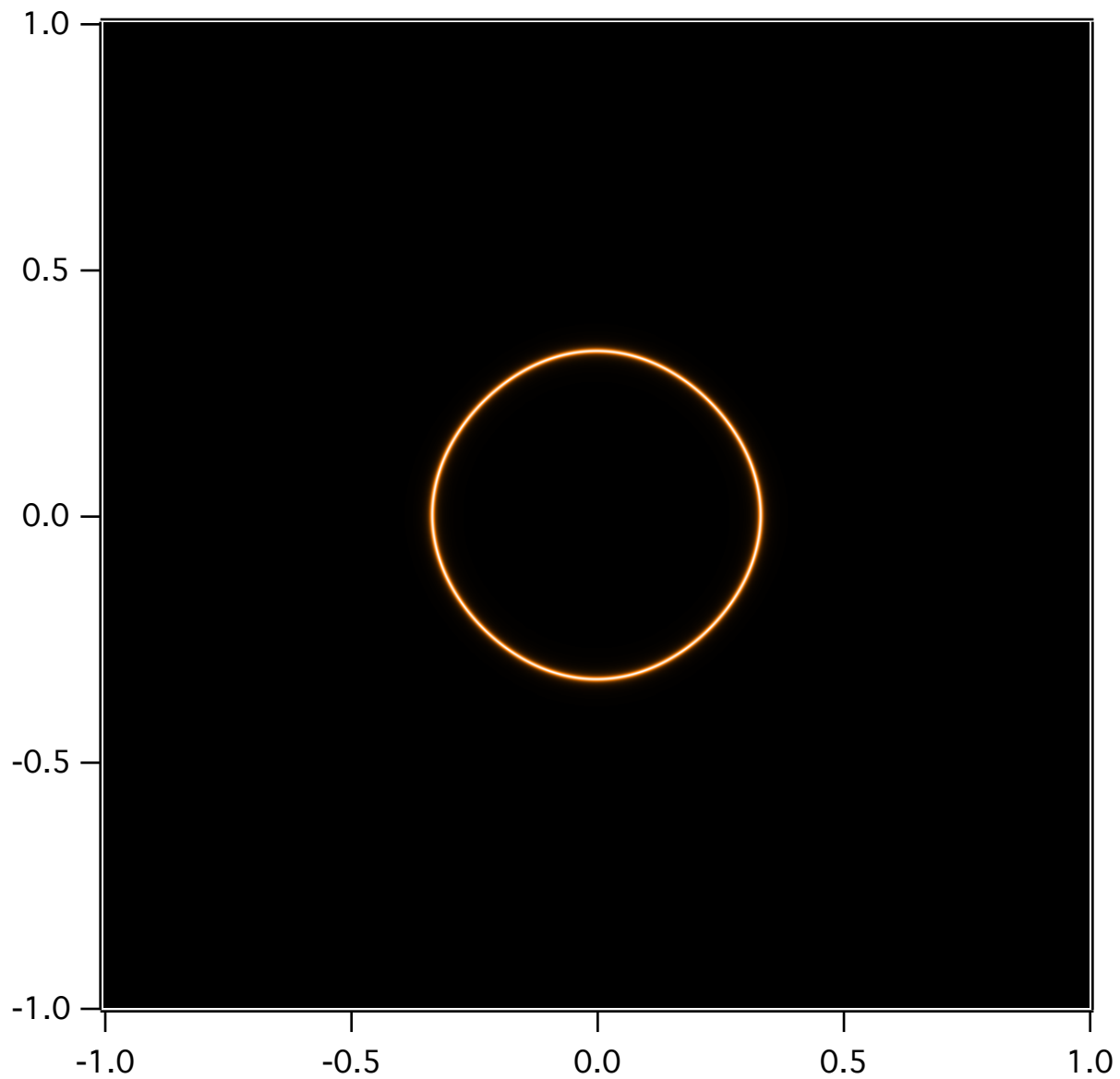
z = -cos(pi*x/2)-cos(pi*y/2)

Number of Gridlines 30

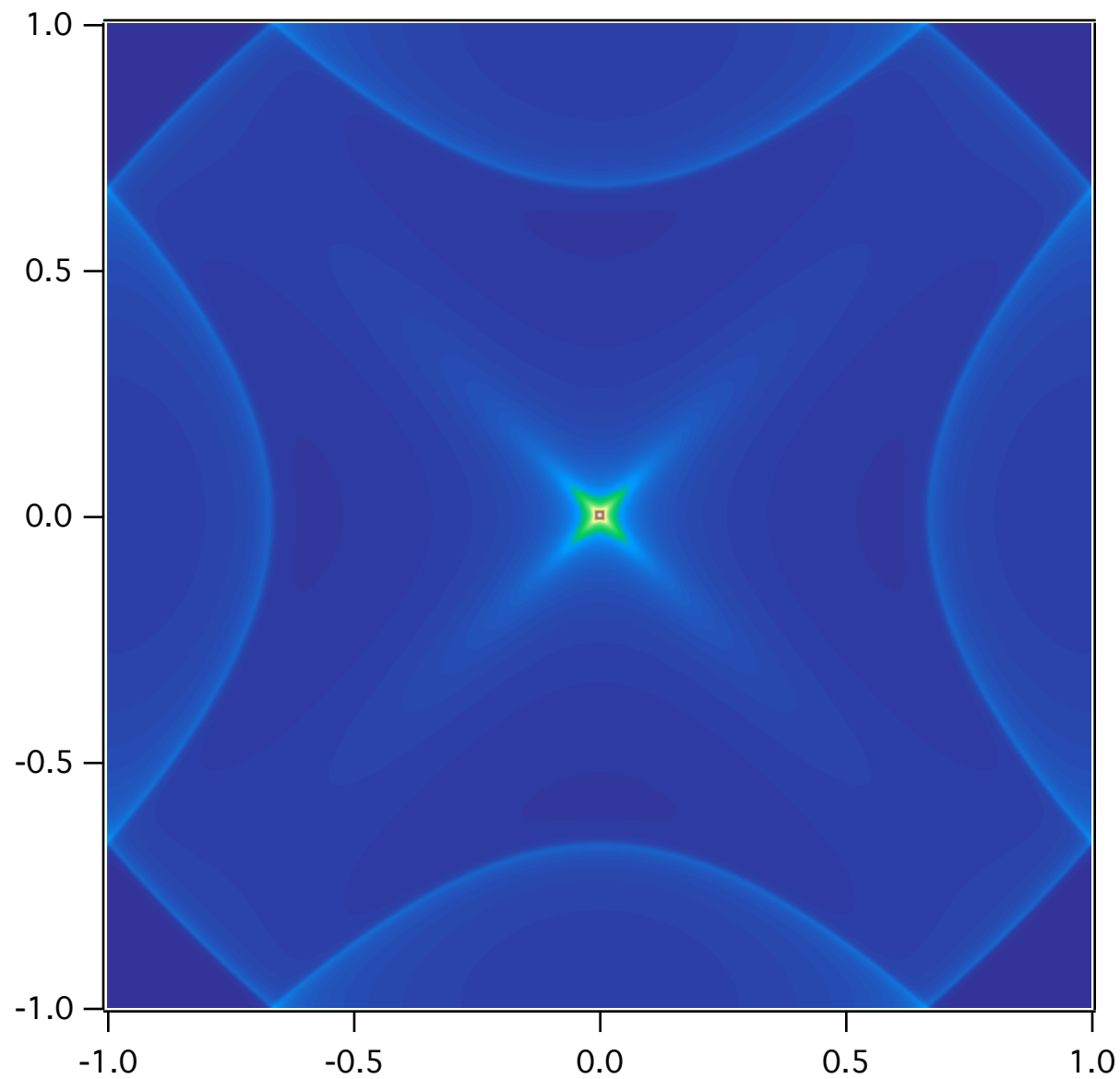
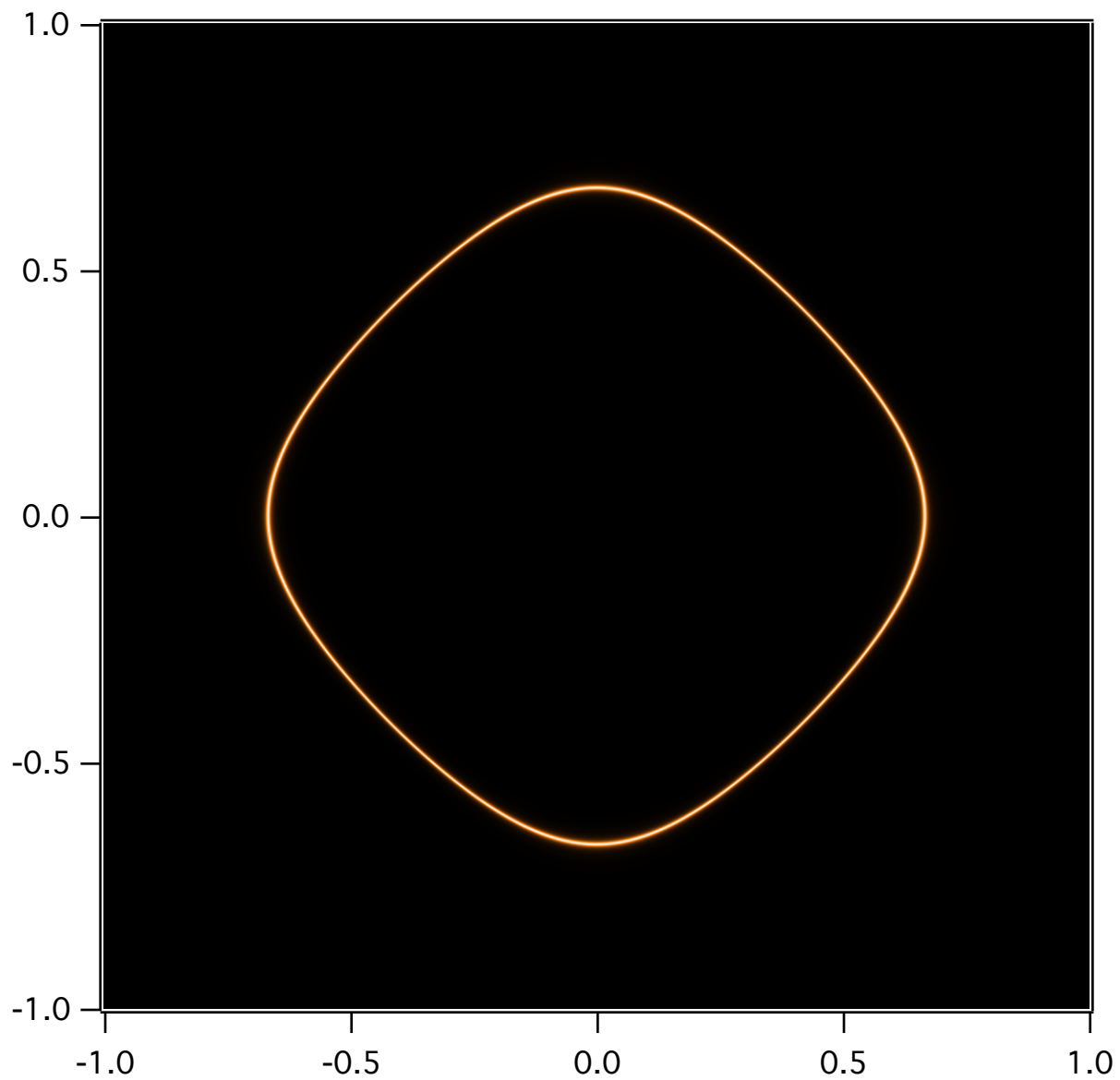
$f(1.9635, 1.9635) = 1.99671288$



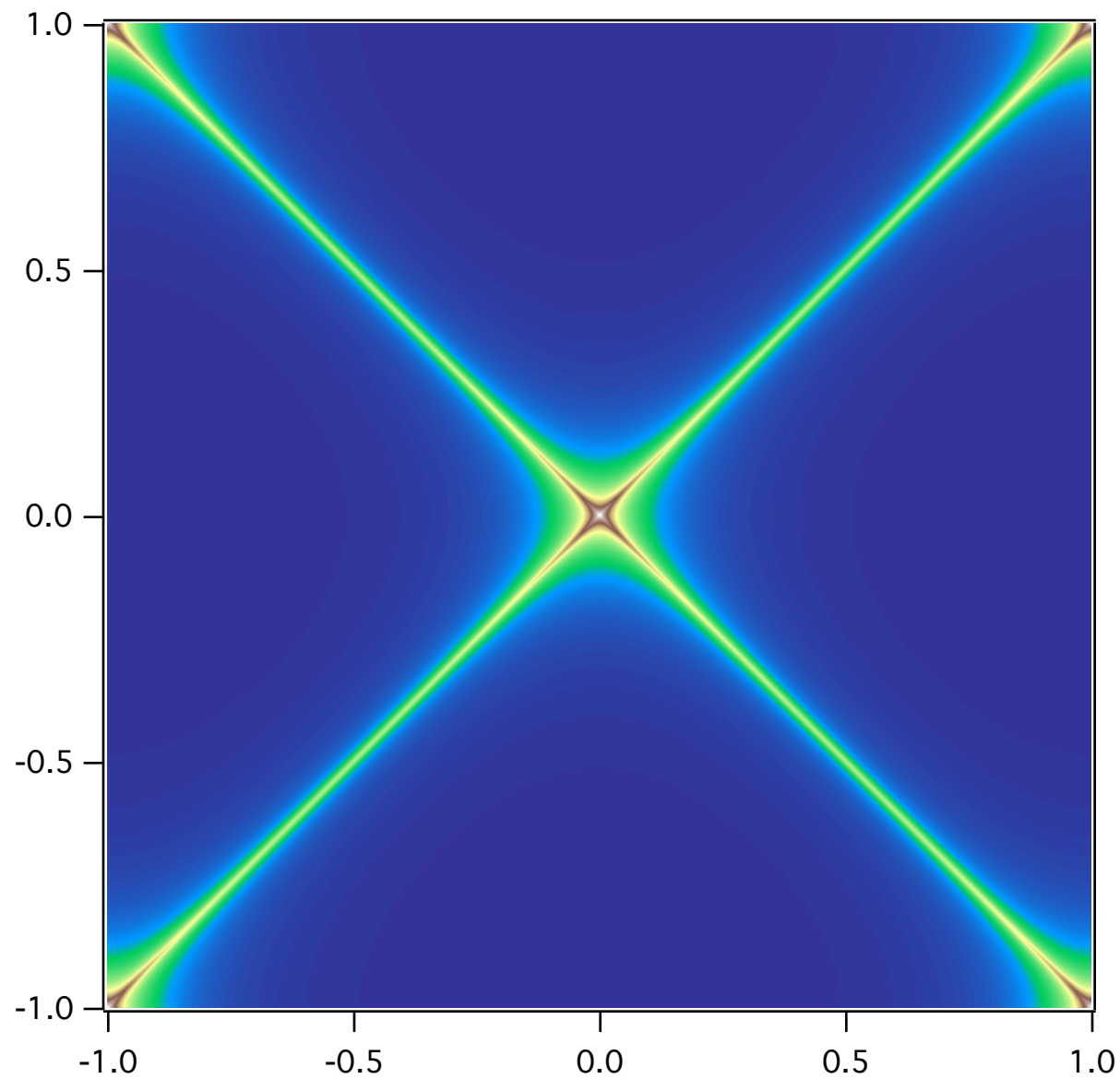
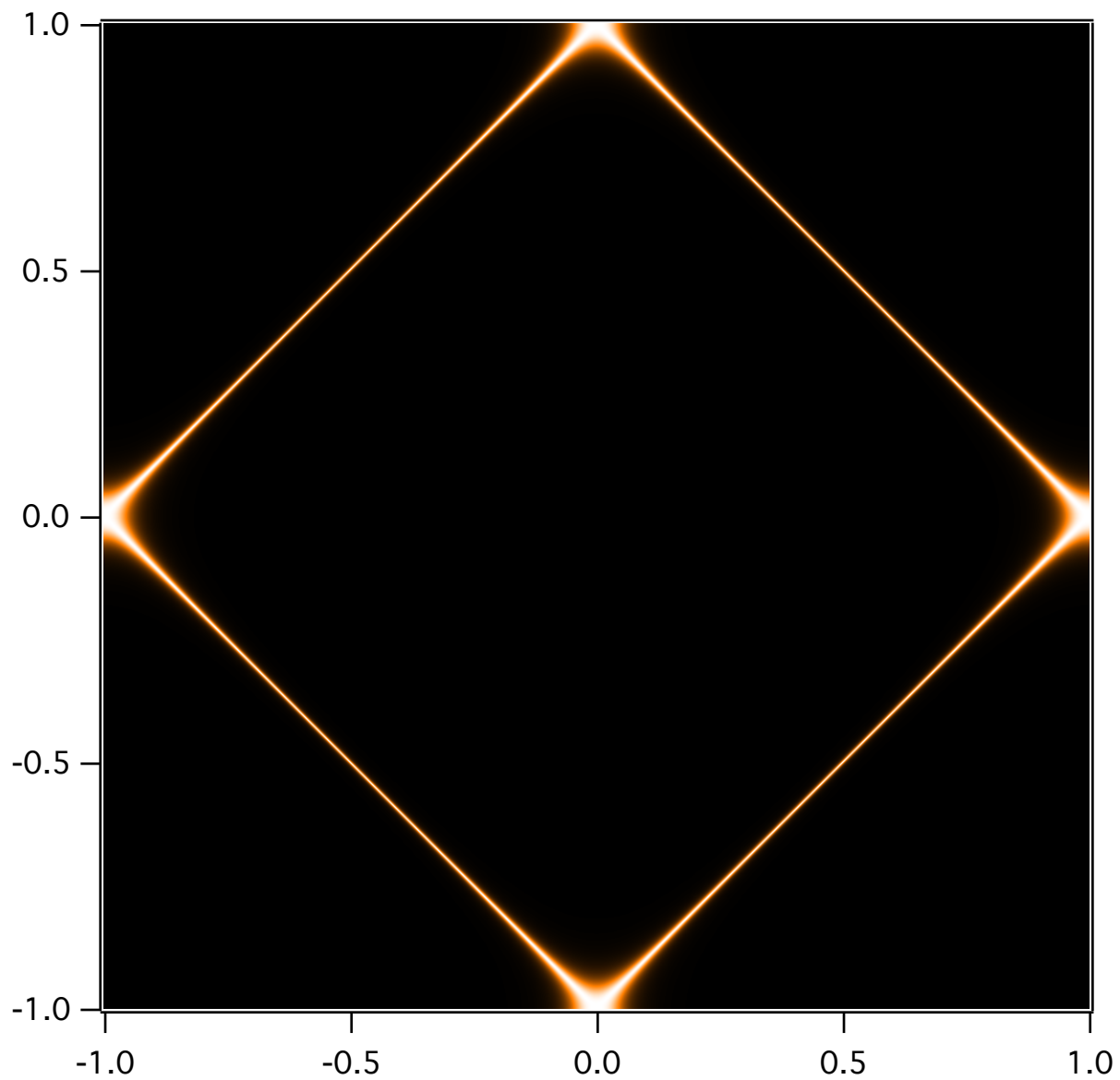
Numerical autocorrelation



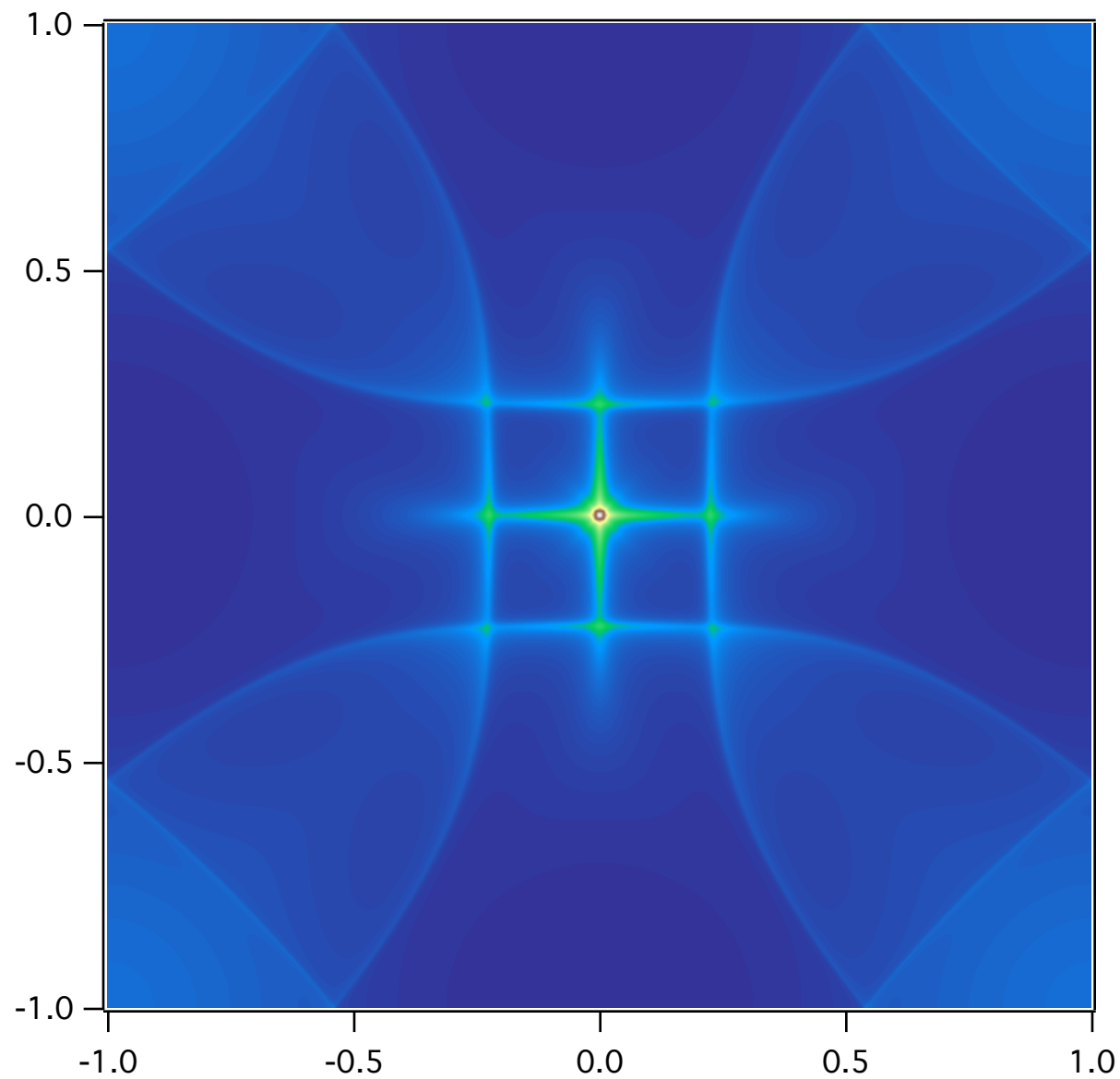
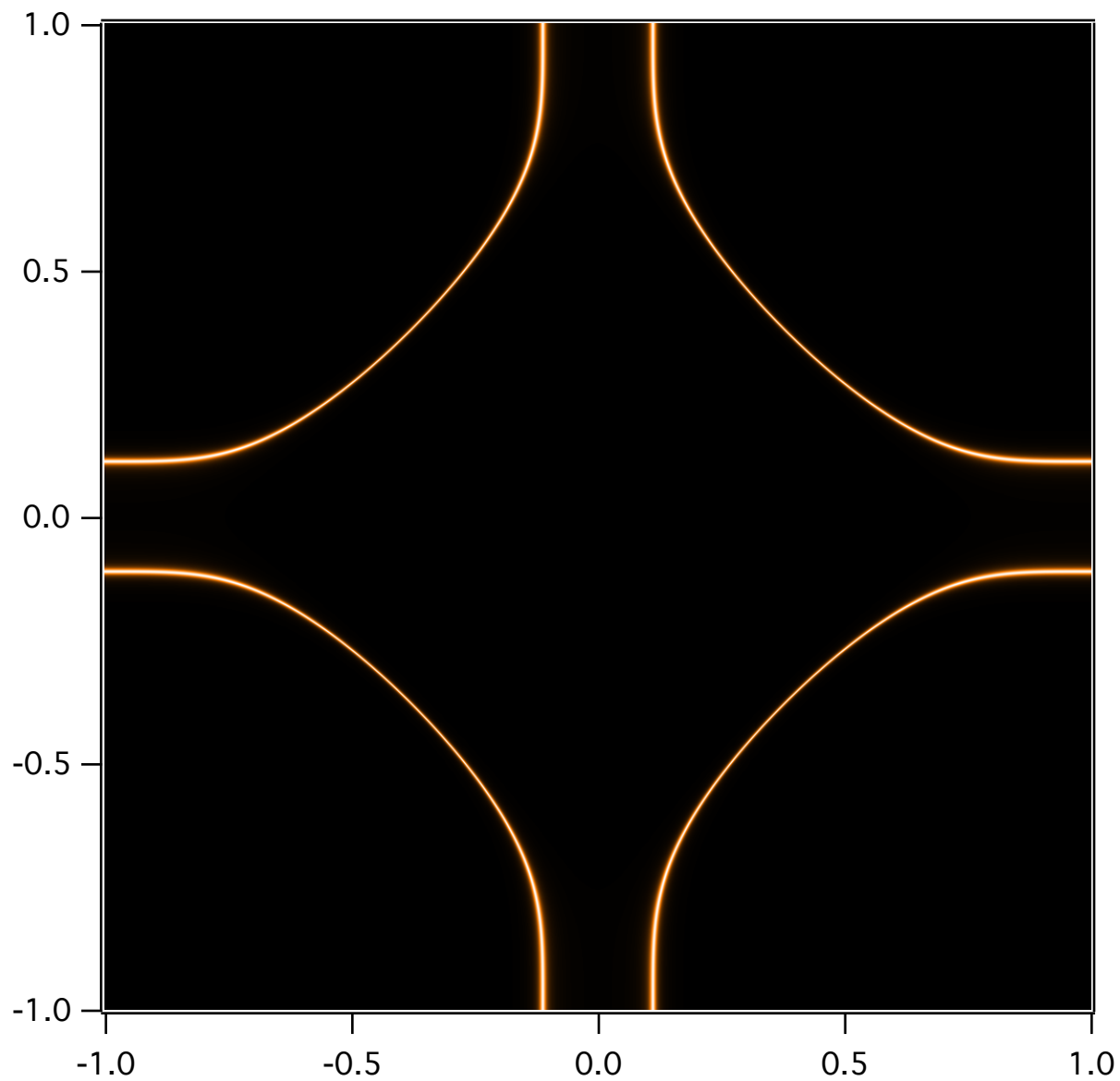
Numerical autocorrelation



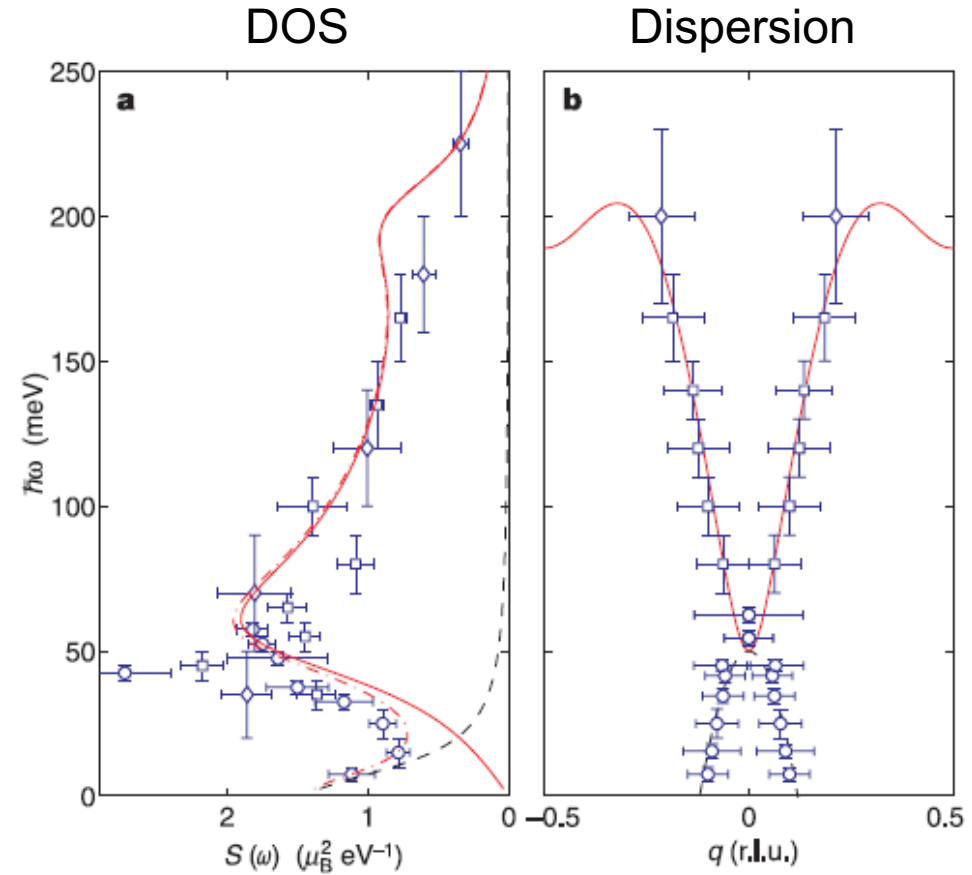
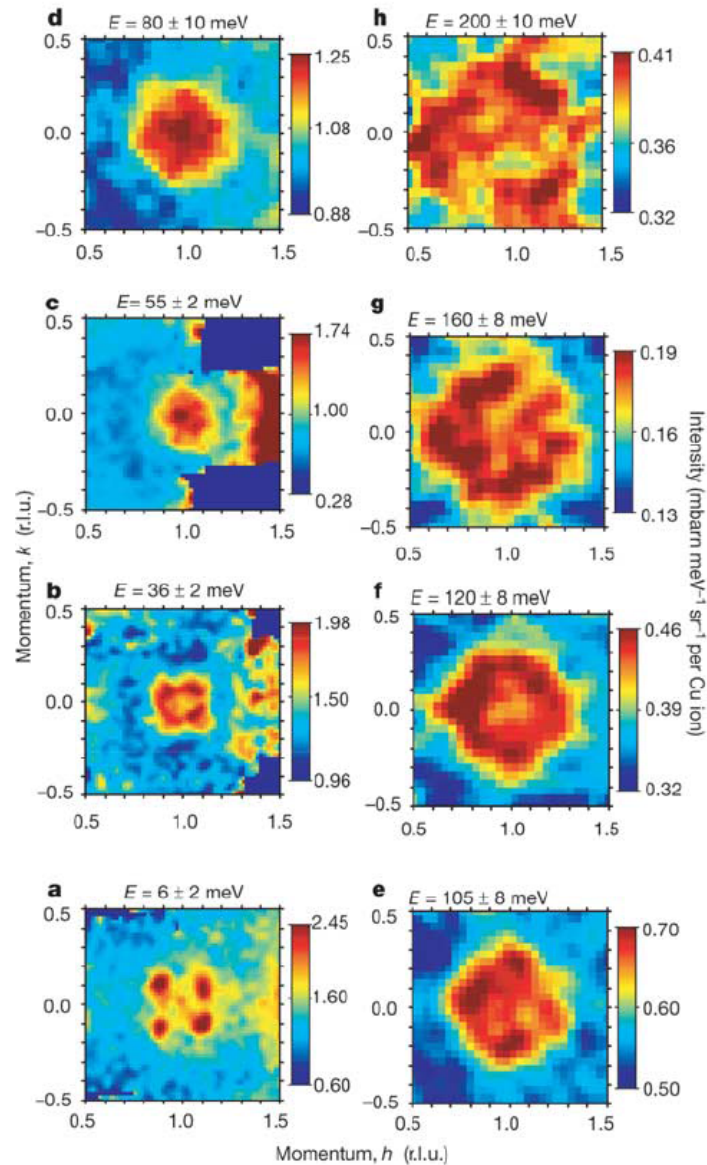
Numerical autocorrelation



Numerical autocorrelation



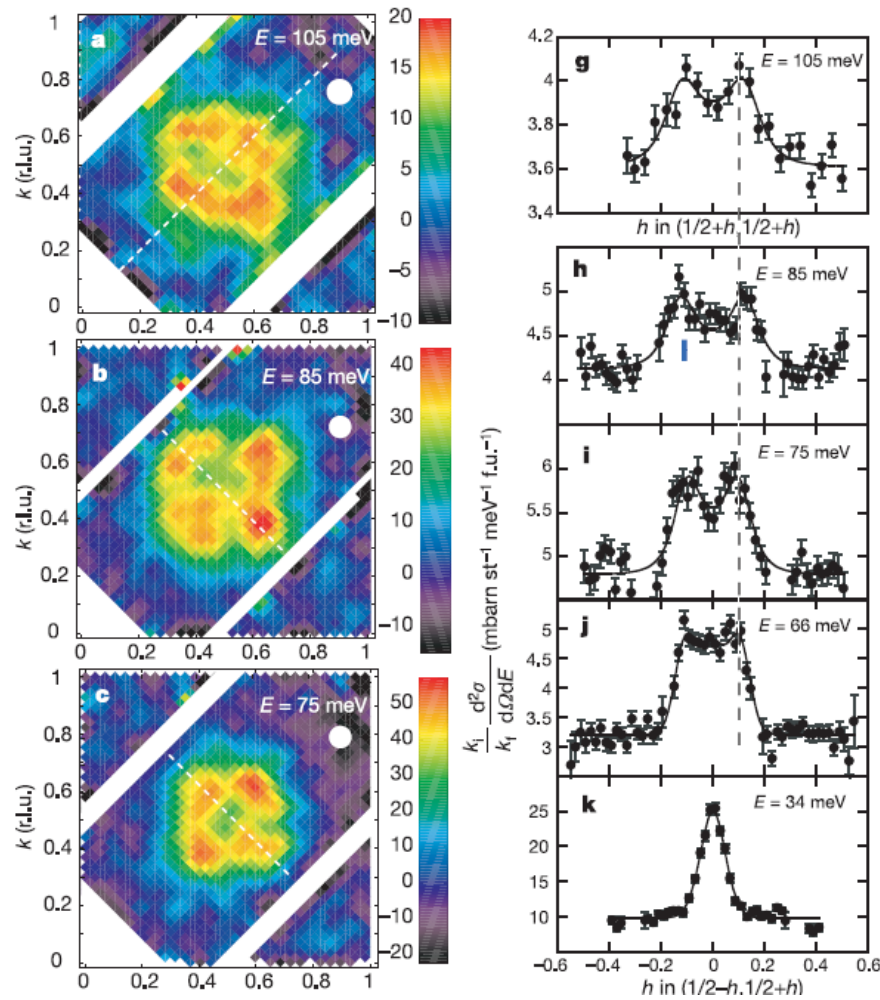
Spin susceptibility structure



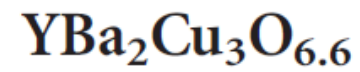
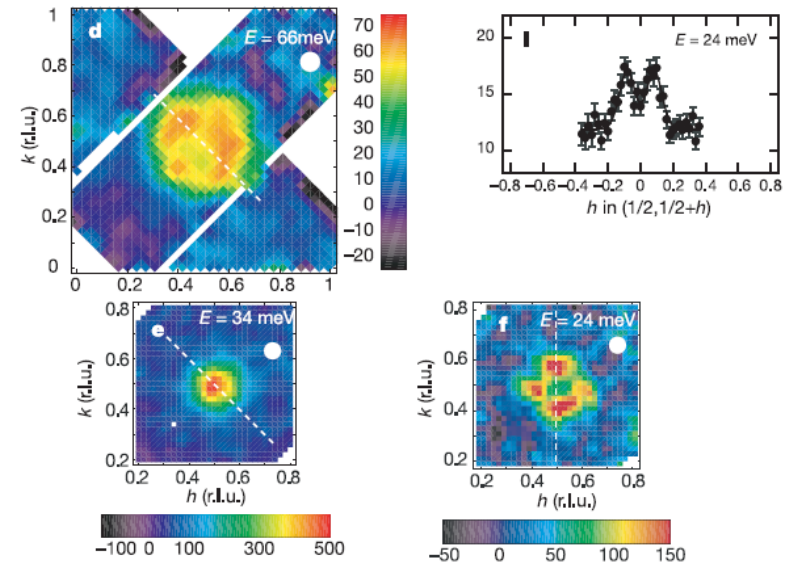
$\text{La}_{2-x}\text{Ba}_x\text{CuO}_4$ ('Zurich' oxide)

Tranquada *Nature* 2004

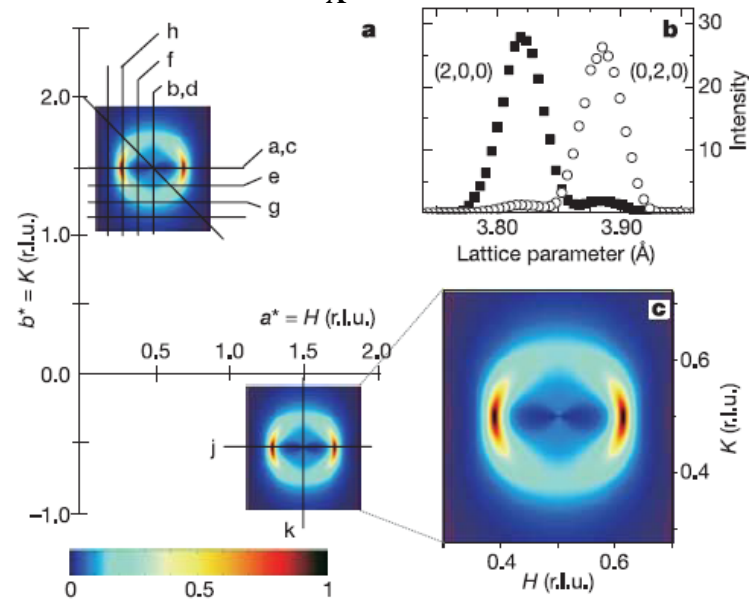
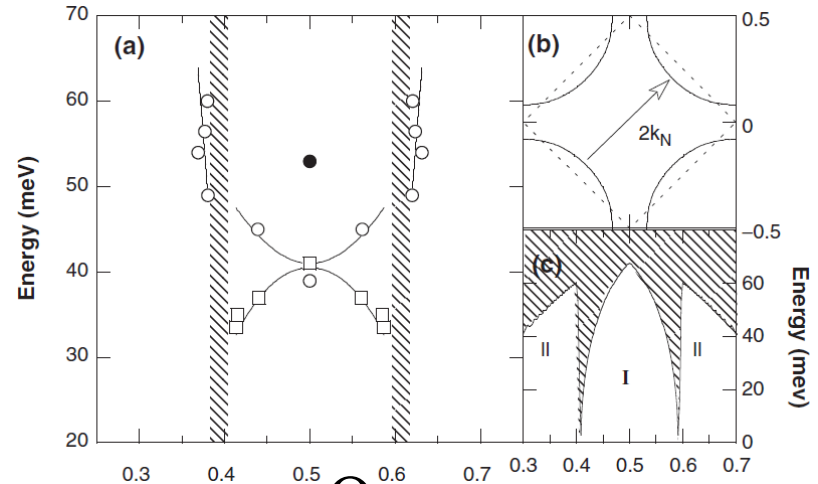
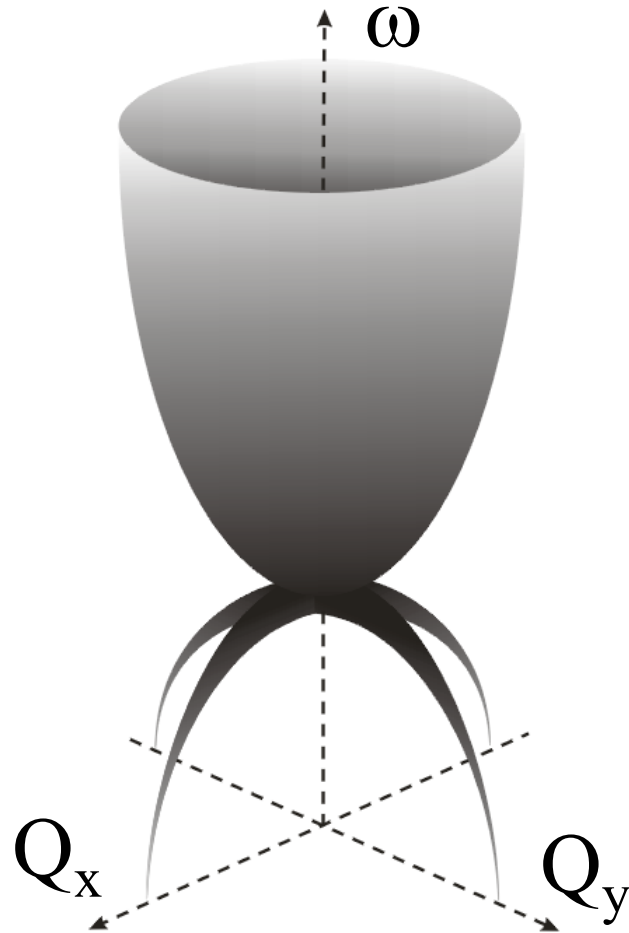
Spin susceptibility structure



$$\chi(\omega, \mathbf{Q})$$



Spin susceptibility structure



$\text{YBa}_2\text{Cu}_3\text{O}_{6.6}$

Looking for "fingerprints"

if 2nd order perturbation theory works

$$(\Delta, \Sigma) = \text{EE}(\Delta, \Sigma, \varepsilon, \chi) \quad \text{SC}$$

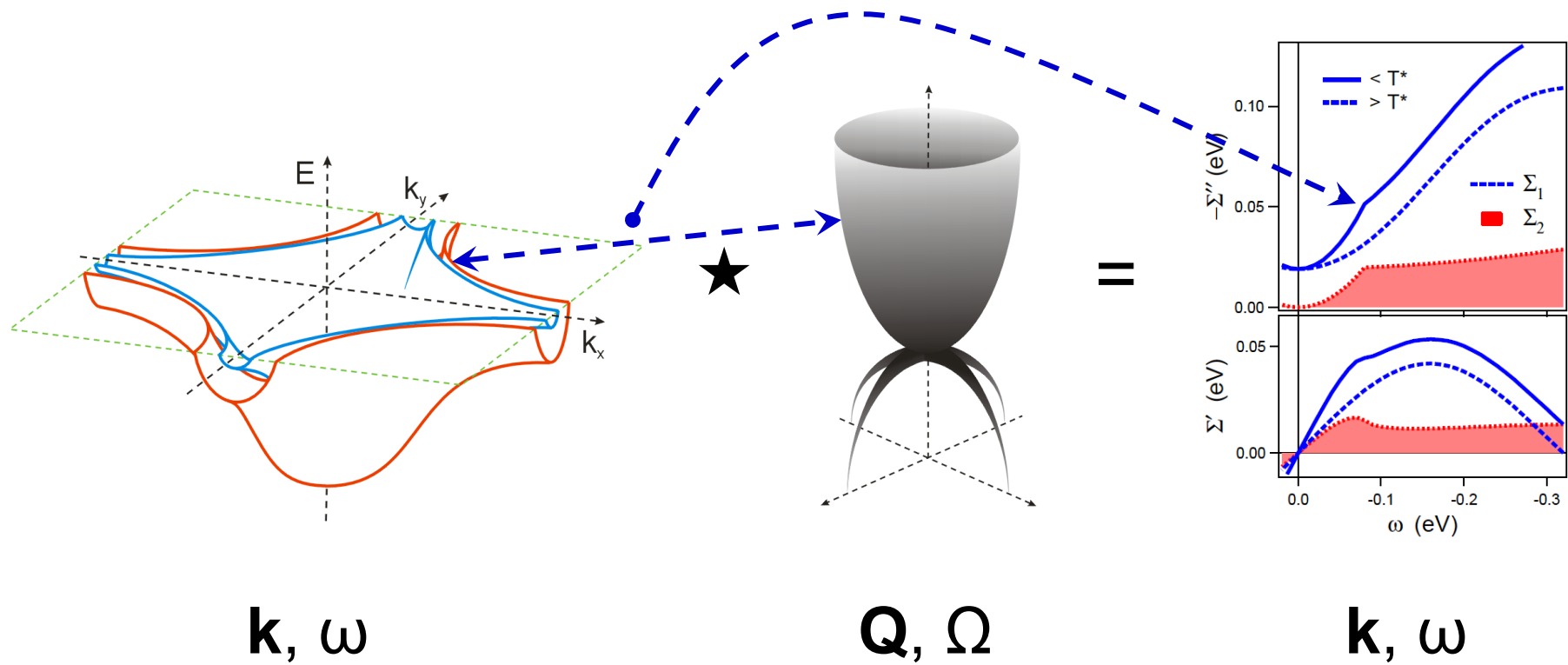
$$\Sigma \sim (G \star \chi)_{\mathbf{k}, \omega} \quad \text{N}$$

$$\chi_{\text{it}} \sim (G \star G)_{\mathbf{k}, \omega} \quad \text{"itinerant" magnetism}$$

①

$$G \star \chi \sim \Sigma$$

$$\Sigma(\mathbf{k}, \omega) \sim \int G(\mathbf{k} + \mathbf{Q}, \omega + \Omega) \chi(\mathbf{Q}, \Omega) d\mathbf{Q} d\Omega$$



①

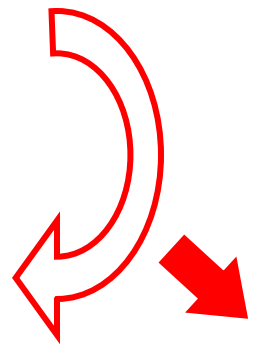
LDA or
ARPES

$$G_0 \star X_{\text{exp}} \sim \Sigma_i$$

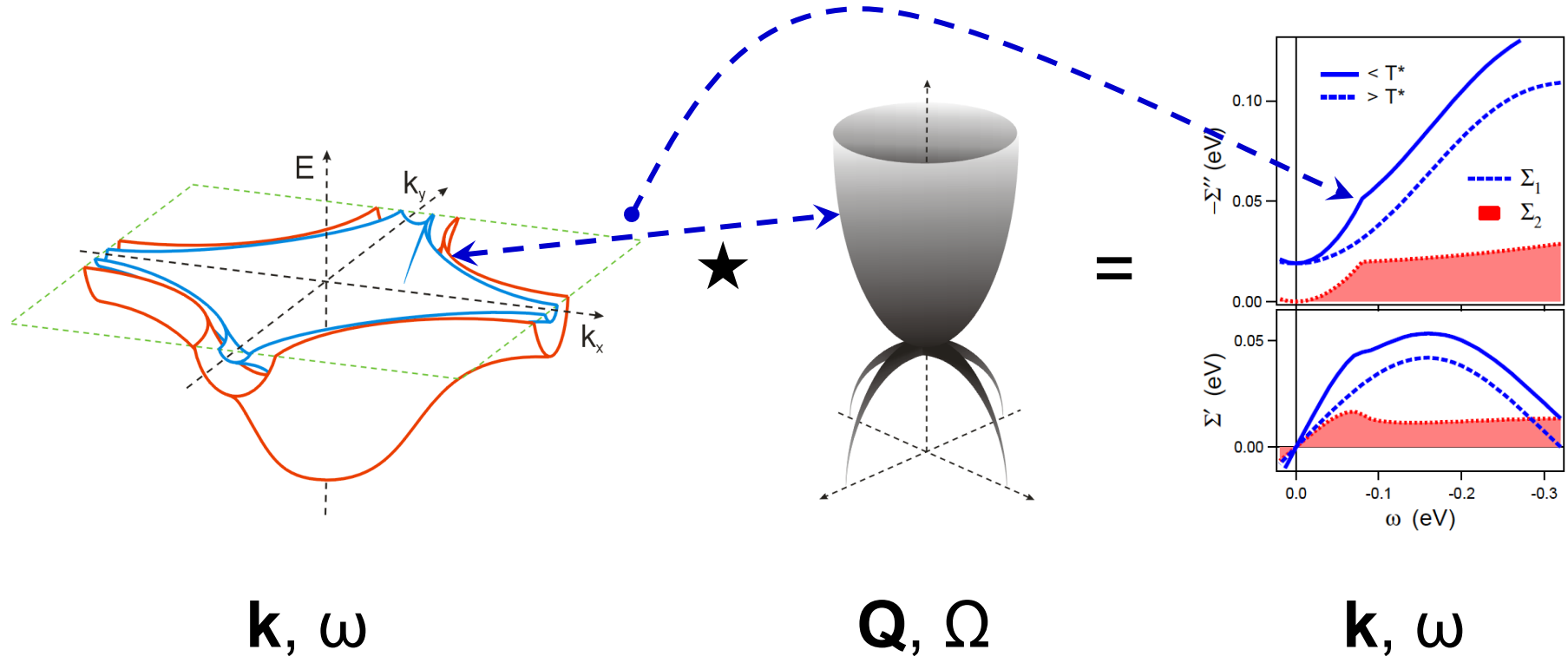


INS

$$G_i^{-1} = G_0^{-1} + \Sigma_i$$



ARPES



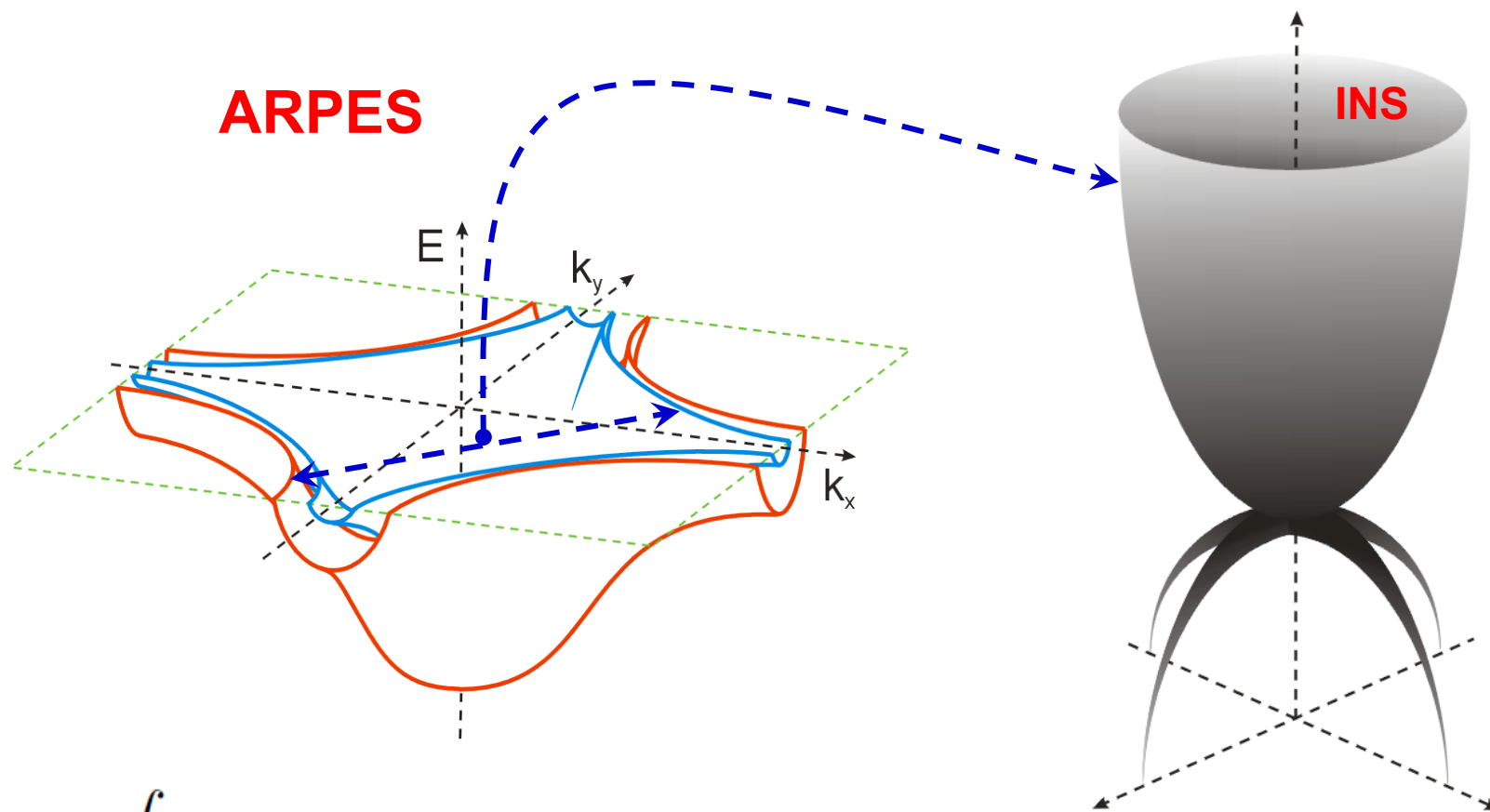
\mathbf{k}, ω

\mathbf{Q}, Ω

\mathbf{k}, ω

itinerant

$$\chi \sim G \star G$$



$$\chi_0(\mathbf{Q}, \Omega) \propto -2i \int G(\mathbf{k}, \omega) G(\mathbf{k} + \mathbf{Q}, \omega + \Omega) d^2k d\omega$$

$$\chi(\mathbf{Q}, \Omega) = \chi_0(\mathbf{Q}, \Omega) / [1 + J_{\mathbf{Q}} \chi_0(\mathbf{Q}, \Omega)]$$

RPA

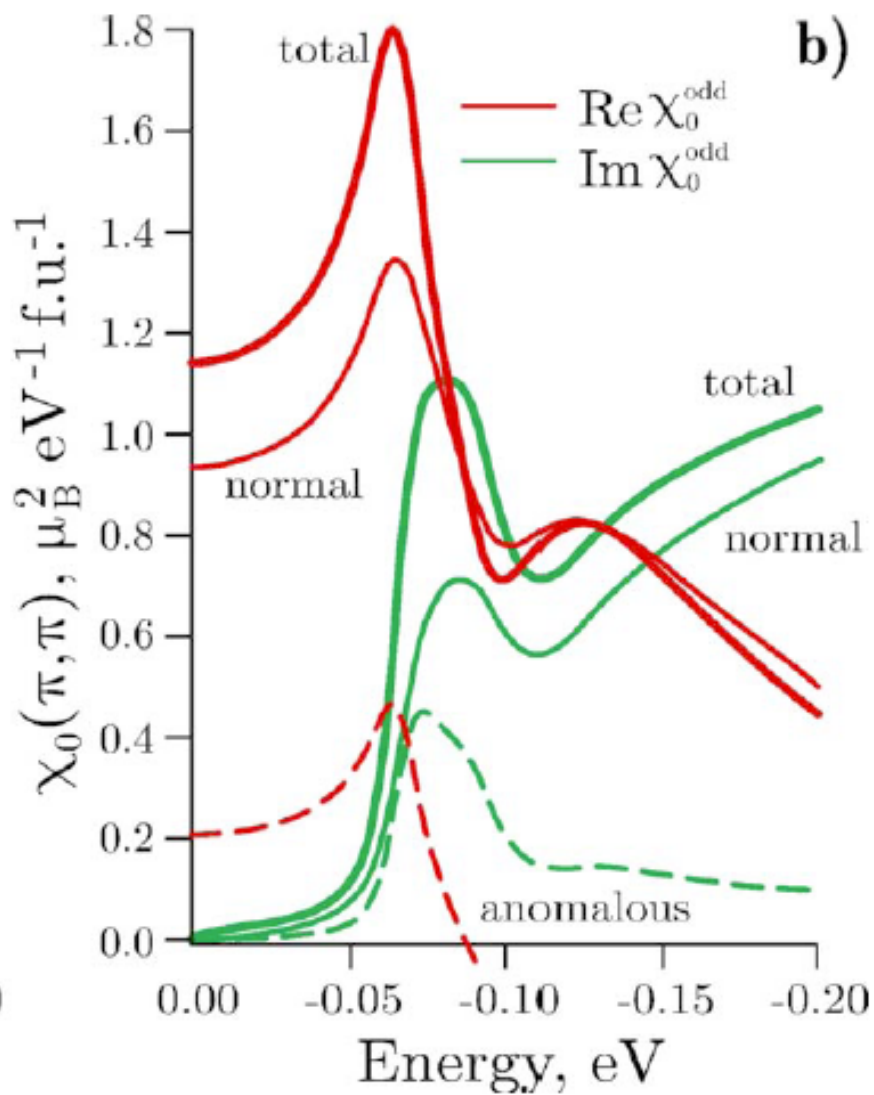
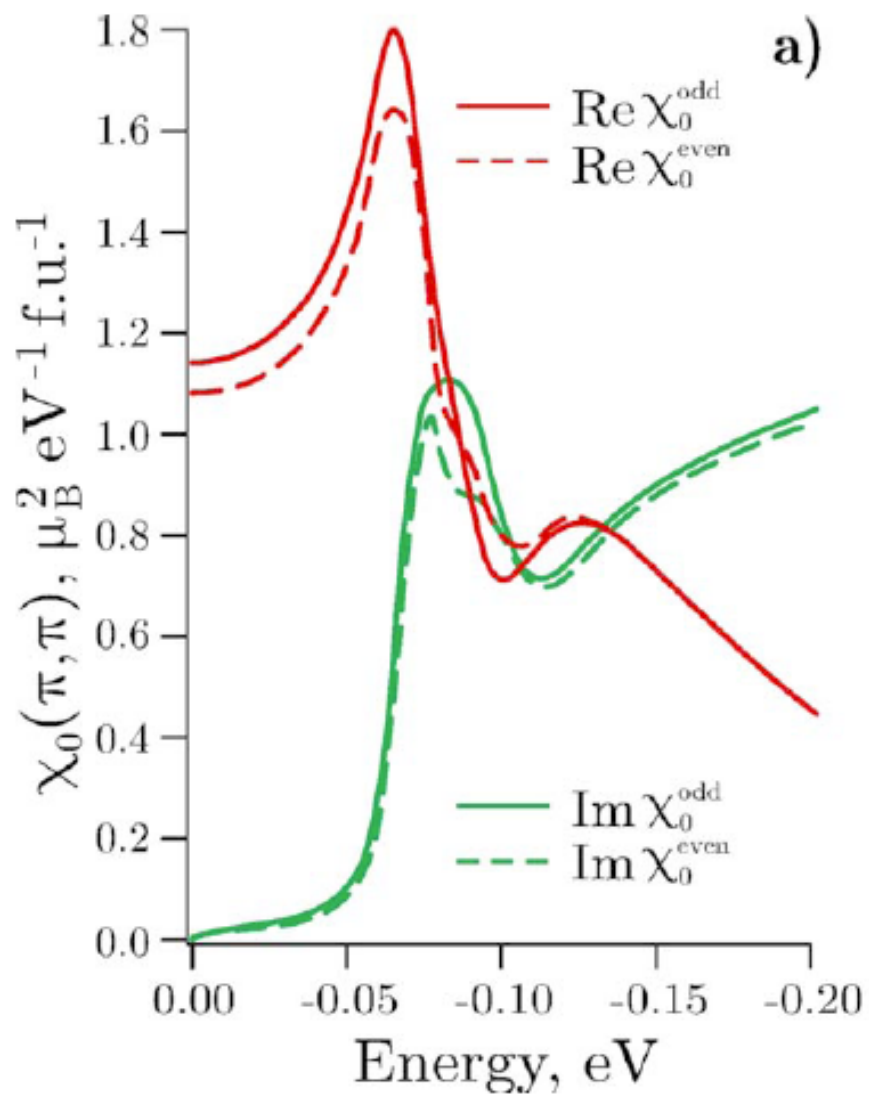
$$\chi \sim G \star G$$

$$\chi_0(\mathbf{Q}, i\Omega_n) = \frac{1}{\pi^2} \int \sum_m G(\mathbf{k}, i\omega_m) G(\mathbf{k} + \mathbf{Q}, i\omega_m + i\Omega_n) d\mathbf{k}$$

$$\chi_0^{o,e}(\mathbf{Q}, \Omega) = \sum_{\substack{i=j(o) \\ i \neq i(e)}} \iint_{-\infty}^{+\infty} C_{ij}(\mathbf{k}, \epsilon, \nu) \frac{n_f(\nu) - n_f(\epsilon)}{\Omega + \nu - \epsilon + i\Gamma} d\nu d\epsilon$$

where

$$C_{ij}(\mathbf{k}, \epsilon, \nu) = \frac{1}{\pi^2} \int [\text{Im } G_i(\mathbf{k}, \epsilon) \text{Im } G_j(\mathbf{k} + \mathbf{Q}, \nu) + \text{Im } F_i(\mathbf{k}, \epsilon) \text{Im } F_j(\mathbf{k} + \mathbf{Q}, \nu)] d\mathbf{k}.$$



$$\chi^{o,e}(\mathbf{Q}, \Omega) = \chi_0^{o,e}(\mathbf{Q}, \Omega) / [1 - J_Q^{o,e} \chi_0^{o,e}(\mathbf{Q}, \Omega)]$$

