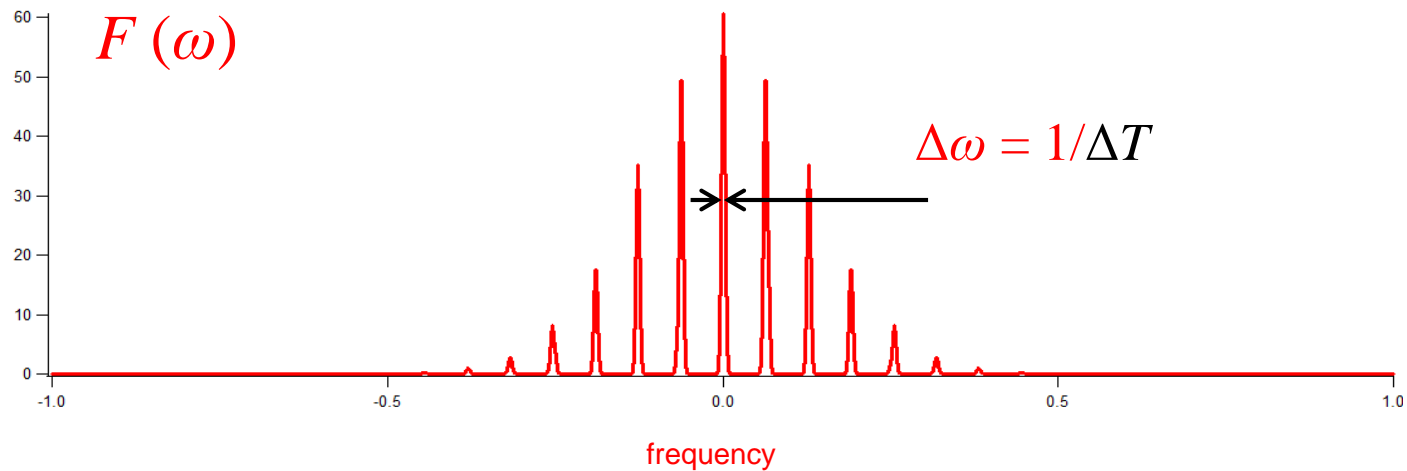
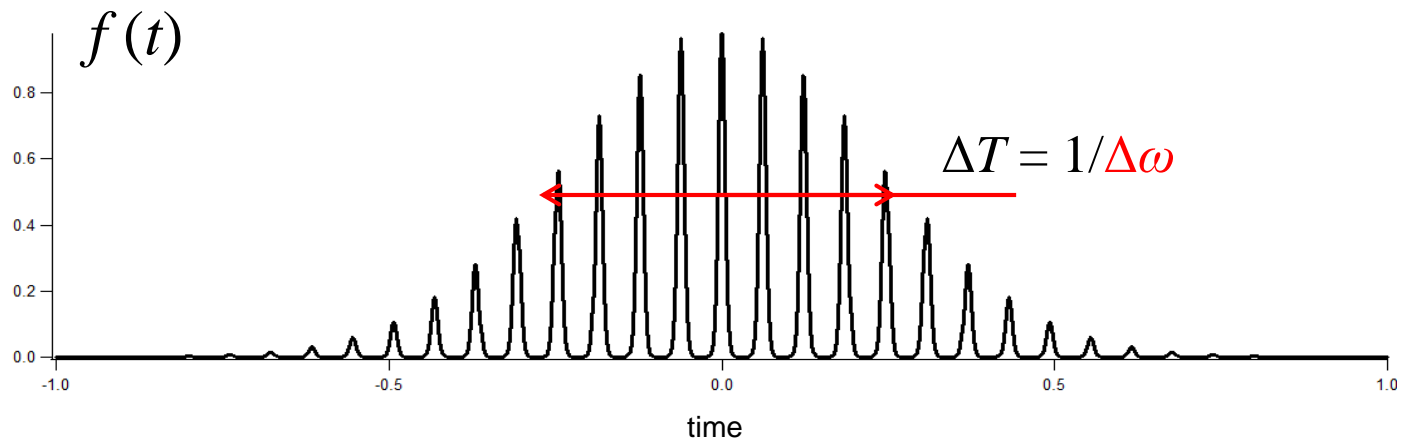
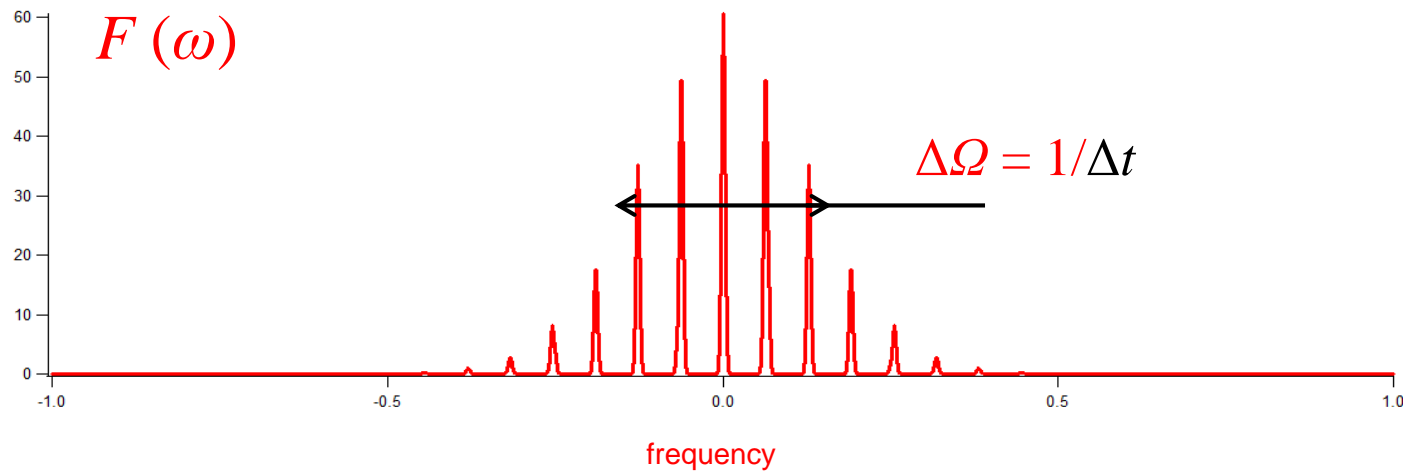
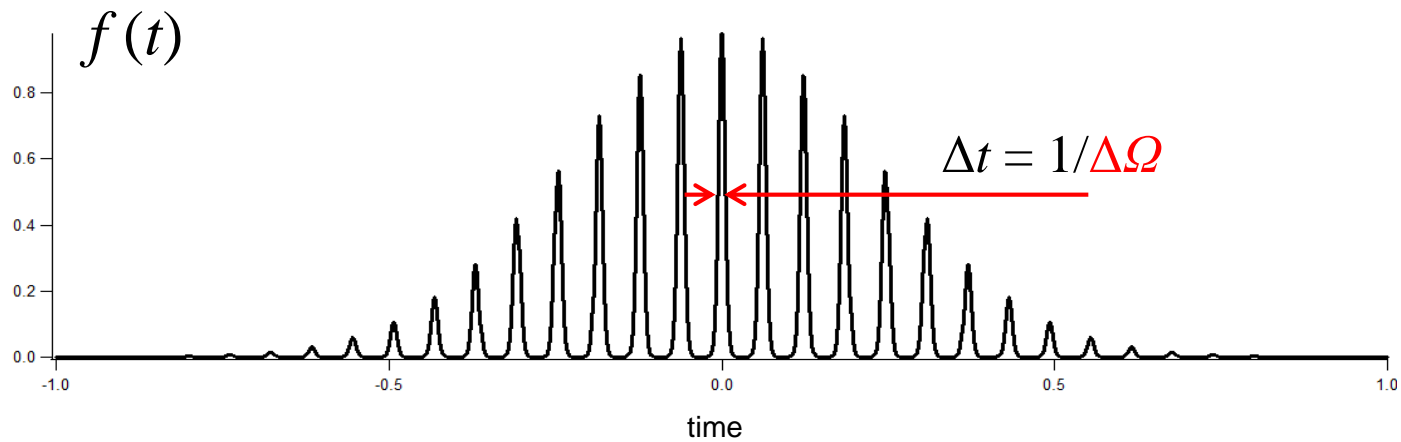
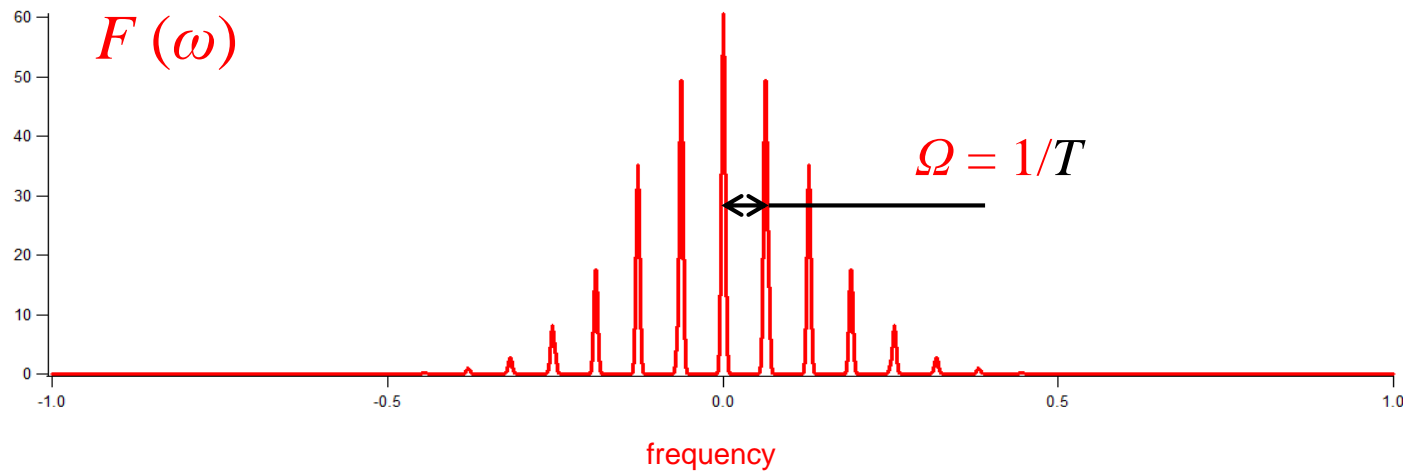
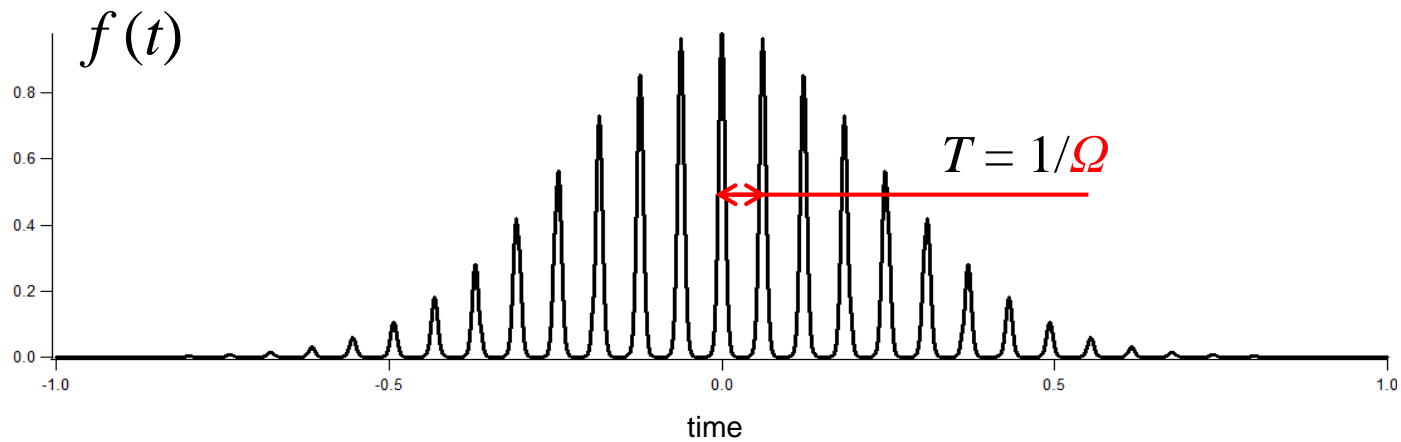


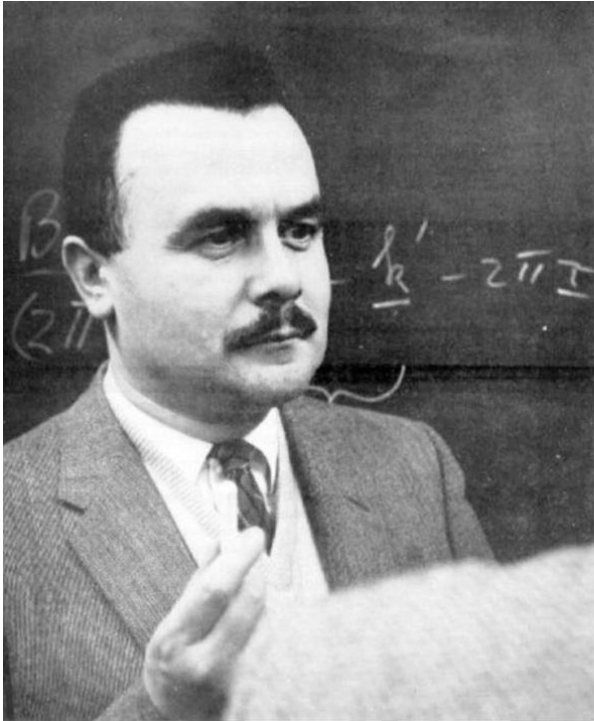
FFT







# Inelastic neutron scattering

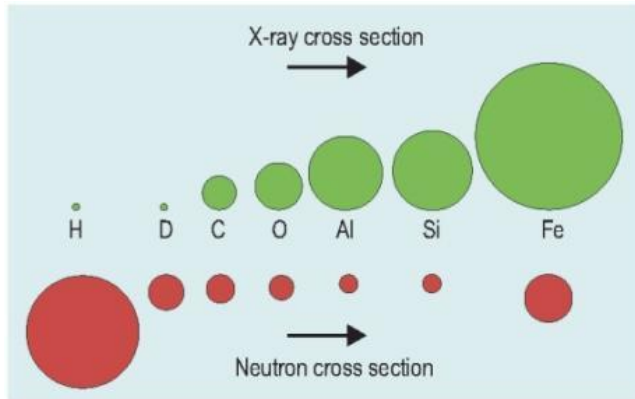
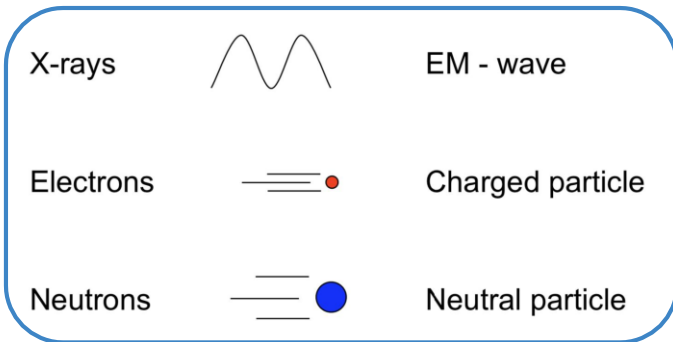


“If the neutron did not exist, it would need to be invented.”

Bertram Brockhouse  
1994 Nobel Laureate in Physics

**Peter Gehring (NIST)**

[https://www.ncnr.nist.gov/summerschool/ss12/pdf/SS2012\\_Gehring\\_InelasticScattering.pdf](https://www.ncnr.nist.gov/summerschool/ss12/pdf/SS2012_Gehring_InelasticScattering.pdf)



### Advantages

Wavelengths easily varied to match atomic spacings

Zero charge → not strongly attenuated by furnaces, etc.

Magnetic dipole moment → can study magnetic structures

Nuclear interaction → can see low-Z elements easily like H → good for the study of biomolecules and polymers.

Nuclear interaction is simple → scattering is easy to model  
 Low energies → Non-destructive probe

### Disadvantages

Neutrons are expensive to produce → access is not as easy

Interact weakly with matter → often require large samples

Available fluxes are low compared to those for x-rays

Let's consider neutrons ...

**Peter Gehring (NIST)**

[https://www.ncnr.nist.gov/summerschool/ss12/pdf/SS2012\\_Gehring\\_InelasticScattering.pdf](https://www.ncnr.nist.gov/summerschool/ss12/pdf/SS2012_Gehring_InelasticScattering.pdf)



$$m_n = 1.675 \times 10^{-27} \text{ kg}$$

$$Q = 0$$

$$S = \frac{1}{2} \hbar$$

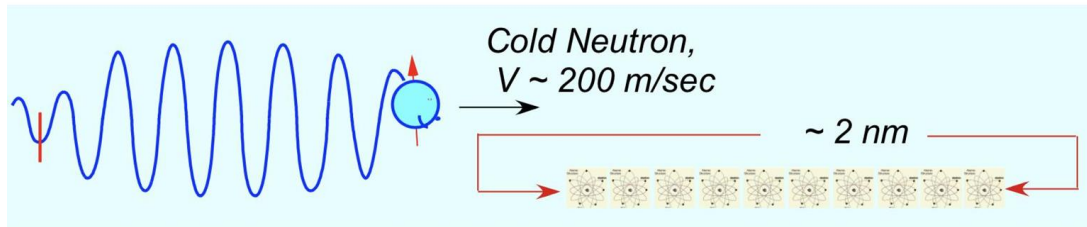
$$\mu_n = -1.913 \mu_N$$



$$\lambda = 1 \text{ \AA}$$

$$v = 4000 \text{ m/s}$$

$$E = 82 \text{ meV}$$



$$\lambda = 9 \text{ \AA}$$

$$v = 440 \text{ m/s}$$

$$E = 1 \text{ meV}$$

Peter Gehring (NIST)



## Neutron Scattering Cross Sections

What are the physical meanings  
of these three cross sections?

$\sigma$  Total # of neutrons scattered per second /  $\Phi_i$ .

$\frac{d\sigma}{d\Omega}$  Total # of neutrons scattered per second into  $d\Omega$  /  $d\Omega \Phi_i$ .  
(**Diffraction** → structure.)

$\frac{d^2\sigma}{d\Omega dE_f}$  Total # of neutrons scattered per second into  $d\Omega$   
with a final energy between  $E_f$  and  $dE_f$  /  $d\Omega dE_f \Phi_i$ .  
(**Inelastic scattering** → dynamics.)

**Peter Gehring (NIST)**

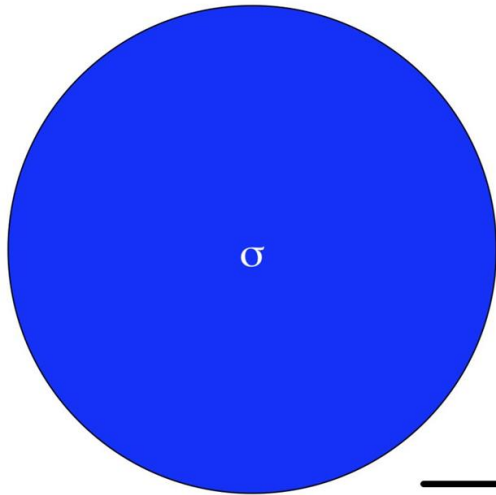
[https://www.ncnr.nist.gov/summerschool/ss12/pdf/SS2012\\_Gehring\\_InelasticScattering.pdf](https://www.ncnr.nist.gov/summerschool/ss12/pdf/SS2012_Gehring_InelasticScattering.pdf)

# Neutron Scattering Cross Sections

What are the relative  
sizes of the cross sections?

Clearly:  $\sigma = \int \frac{d\sigma}{d\Omega} d\Omega = \int \frac{d^2\sigma}{d\Omega dE_f} d\Omega dE_f$

Thus:  $\sigma \gg \frac{d\sigma}{d\Omega} \gg \frac{d^2\sigma}{d\Omega dE_f}$



Typically,  $\frac{d\sigma}{d\Omega} \sim \underline{\underline{10^6}} \times \frac{d^2\sigma}{d\Omega dE_f}$

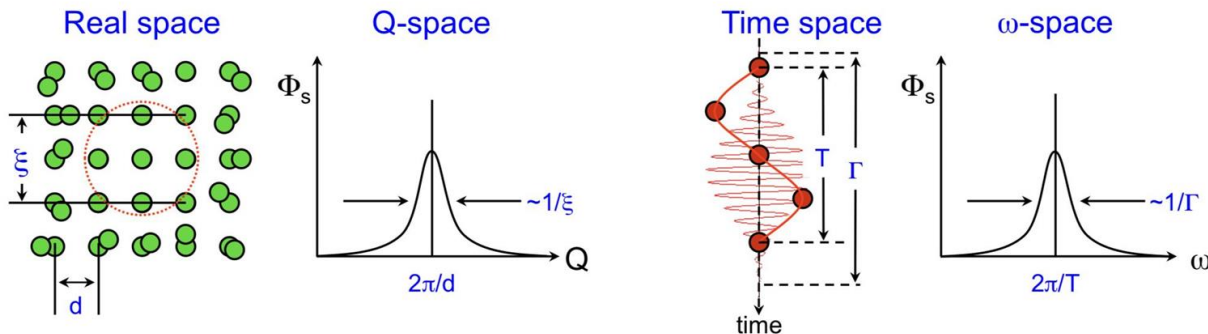
Peter Gehring (NIST)

## Pair Correlation Functions

**KEY IDEA** – Neutron interactions are weak → Scattering only probes two-particle correlations in space and time, but does so simultaneously!

The scattered neutron flux  $\Phi_s(\vec{Q}, \hbar\omega)$  is proportional to the space ( $\vec{r}$ ) and time ( $t$ ) Fourier transform of the probability  $G(\vec{r}, t)$  of finding an atom at  $(\vec{r}, t)$  given that there is another atom at  $r = 0$  at time  $t = 0$ .



$$\Phi_s \propto \frac{\partial^2 \sigma}{\partial \Omega \partial \omega} \propto \iint e^{i(\vec{Q} \cdot \vec{r} - \omega t)} G(\vec{r}, t) d^3 \vec{r} dt$$

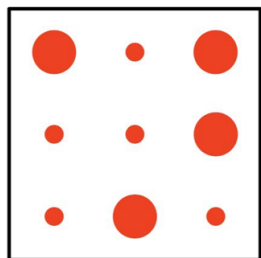


Peter Gehring (NIST)

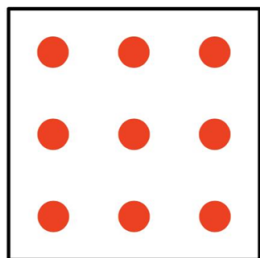
# Neutron Coherent and Incoherent Scattering

Consider a system composed of two different scattering lengths,  $b_1$  and  $b_2$ .

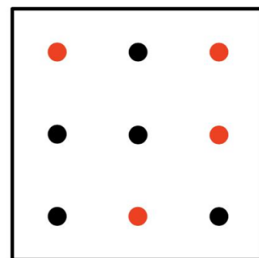
$b_1 =$    
 $b_2 =$  



=



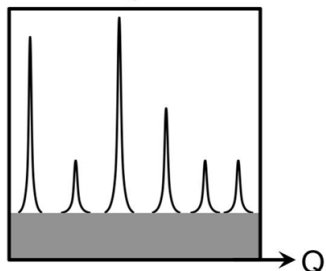
+



The two isotopes are randomly distributed.

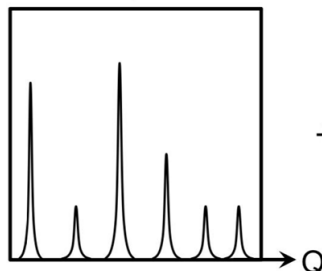
$$\frac{1}{2}(b_1 + b_2) = \bar{b}$$

Deviations  $\delta b$



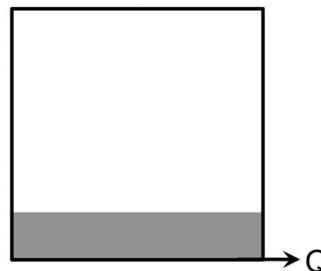
Total scattering

=



Coherent scattering

+



Incoherent scattering

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# Summary of Cross Sections

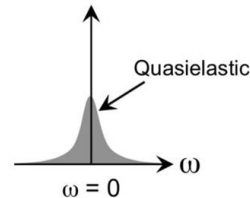
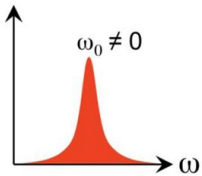
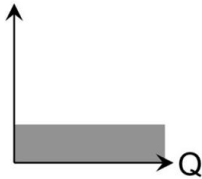
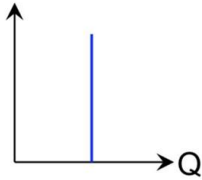
$$\left. \frac{d\sigma}{d\Omega} \right|_{\text{coh}} = \frac{\sigma_{\text{coh}}}{4\pi} S(Q)$$

$$\left. \frac{d\sigma}{d\Omega} \right|_{\text{inc}} = \frac{\sigma_{\text{inc}}}{4\pi}$$

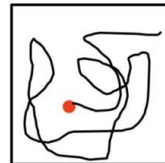
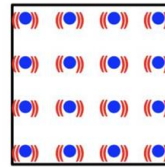
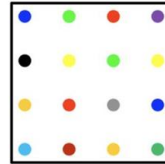
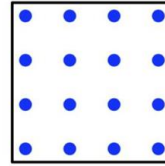
$$\left. \frac{d^2\sigma}{d\Omega dE_f} \right|_{\text{coh}} = \frac{k_f}{k_i} \frac{\sigma_{\text{coh}}}{4\pi} S_{\text{coh}}(Q, \omega)$$

$$\left. \frac{d^2\sigma}{d\Omega dE_f} \right|_{\text{inc}} = \frac{k_f}{k_i} \frac{\sigma_{\text{inc}}}{4\pi} S_{\text{inc}}(Q, \omega)$$

(Q, ω) Space

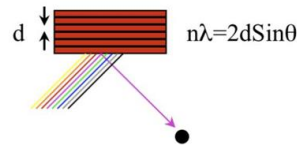


(r,t) Space

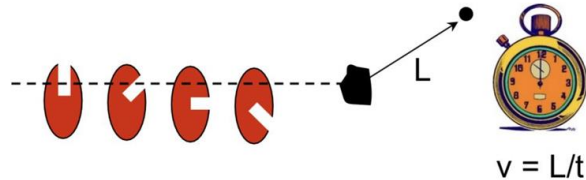


Peter Gehring (NIST)

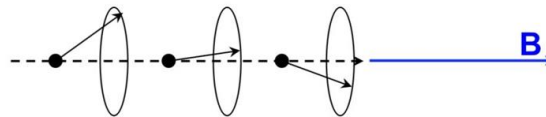
### 1. Bragg Diffraction

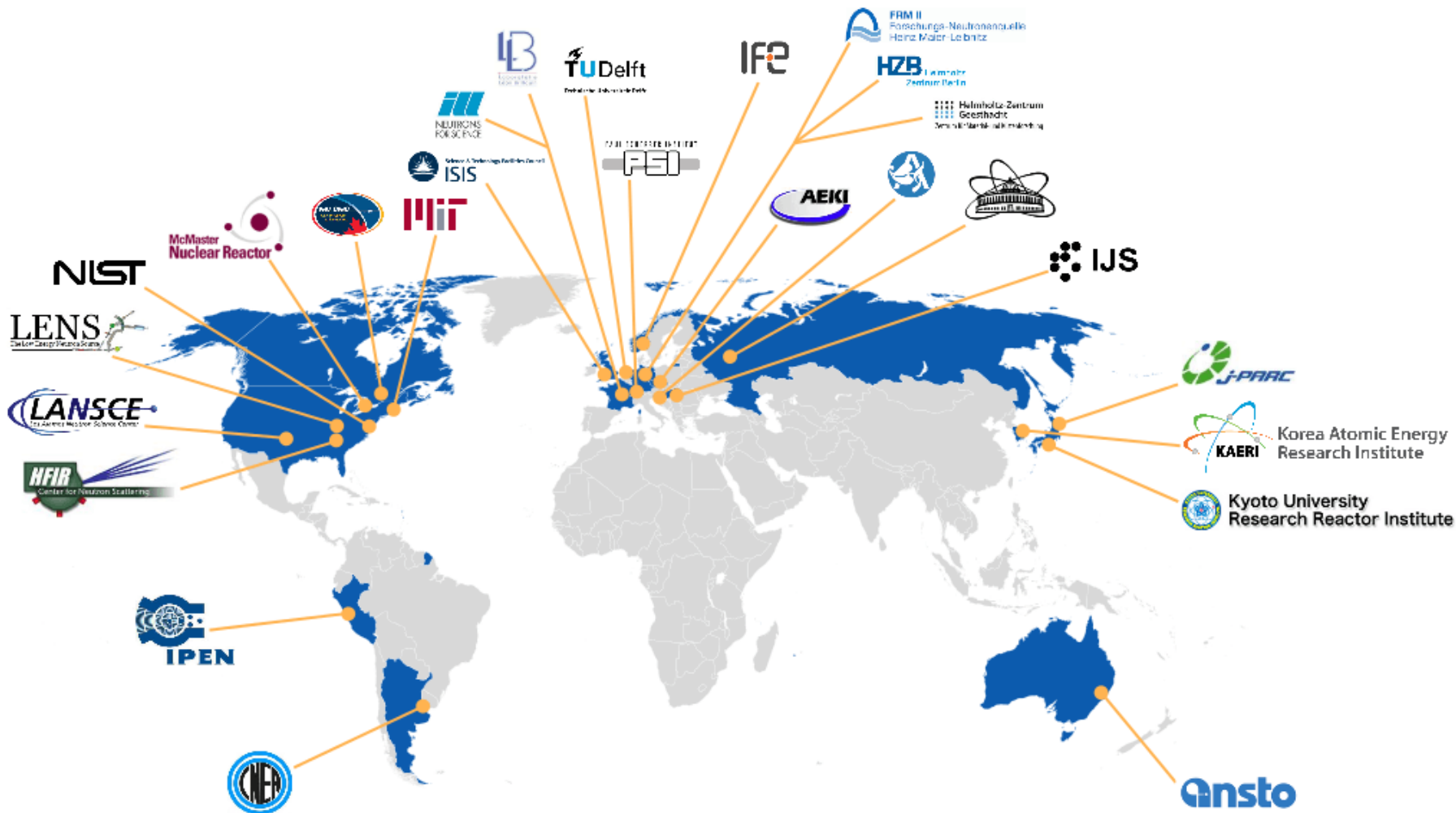


### 2. Time-of-Flight (TOF)

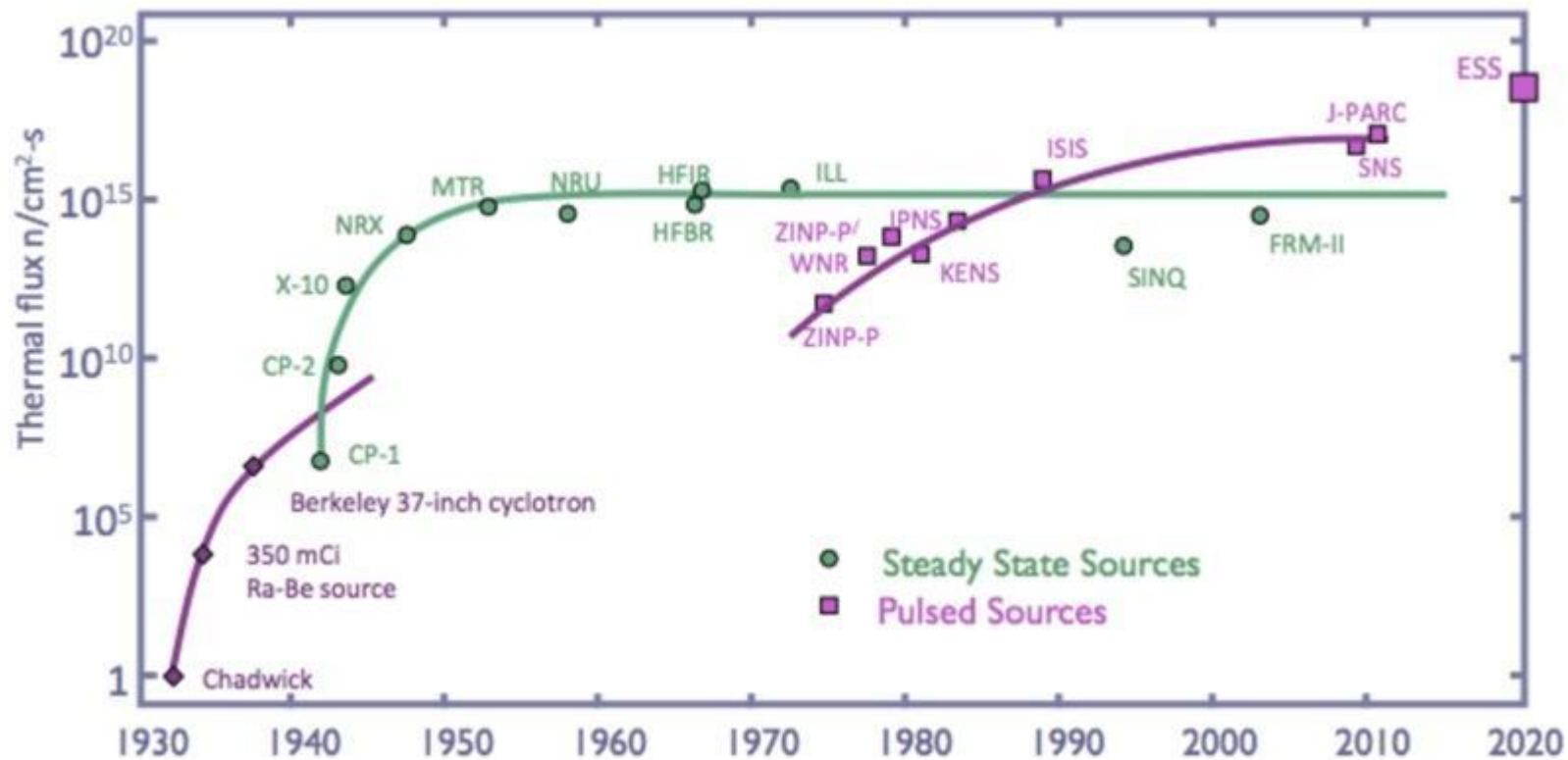


### 3. Larmor Precession

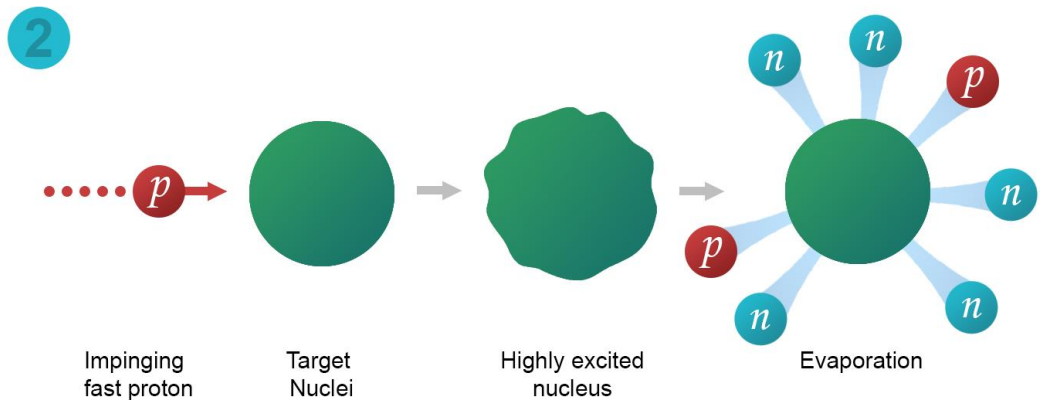
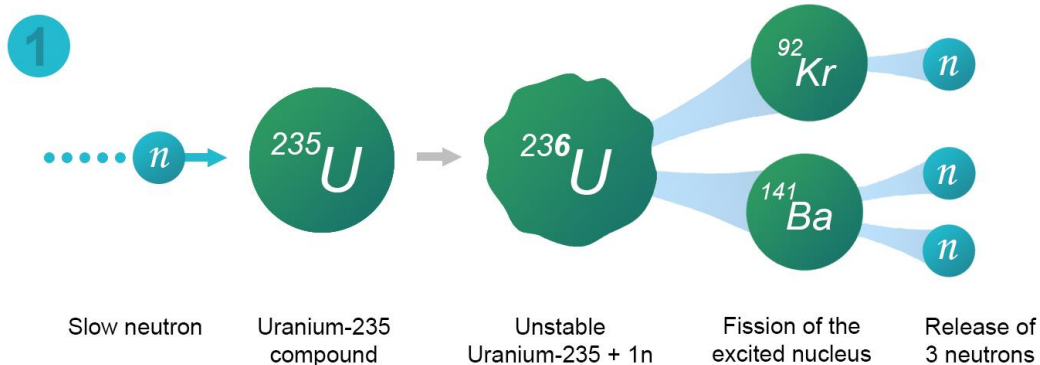




# Evolution of neutron sources







- **Fission:** A high continuous flux of neutrons is produced in the core of a conventional fission reactor.
- **Spallation:** A pulsed production of neutrons is obtained by bombarding a target of heavy elements with high-energy particles, typically accelerated protons.

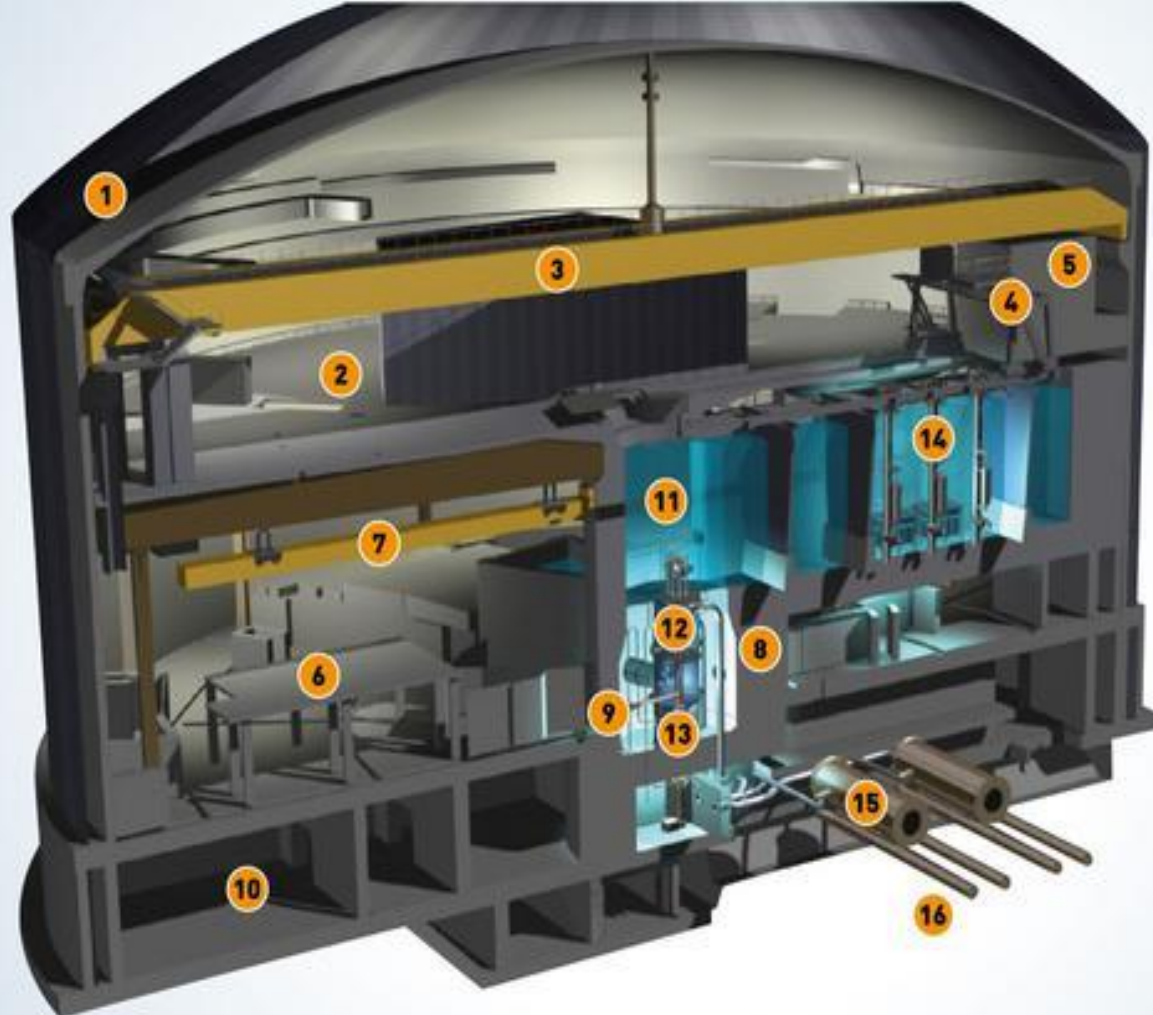


NEUTRONS  
FOR SCIENCE®

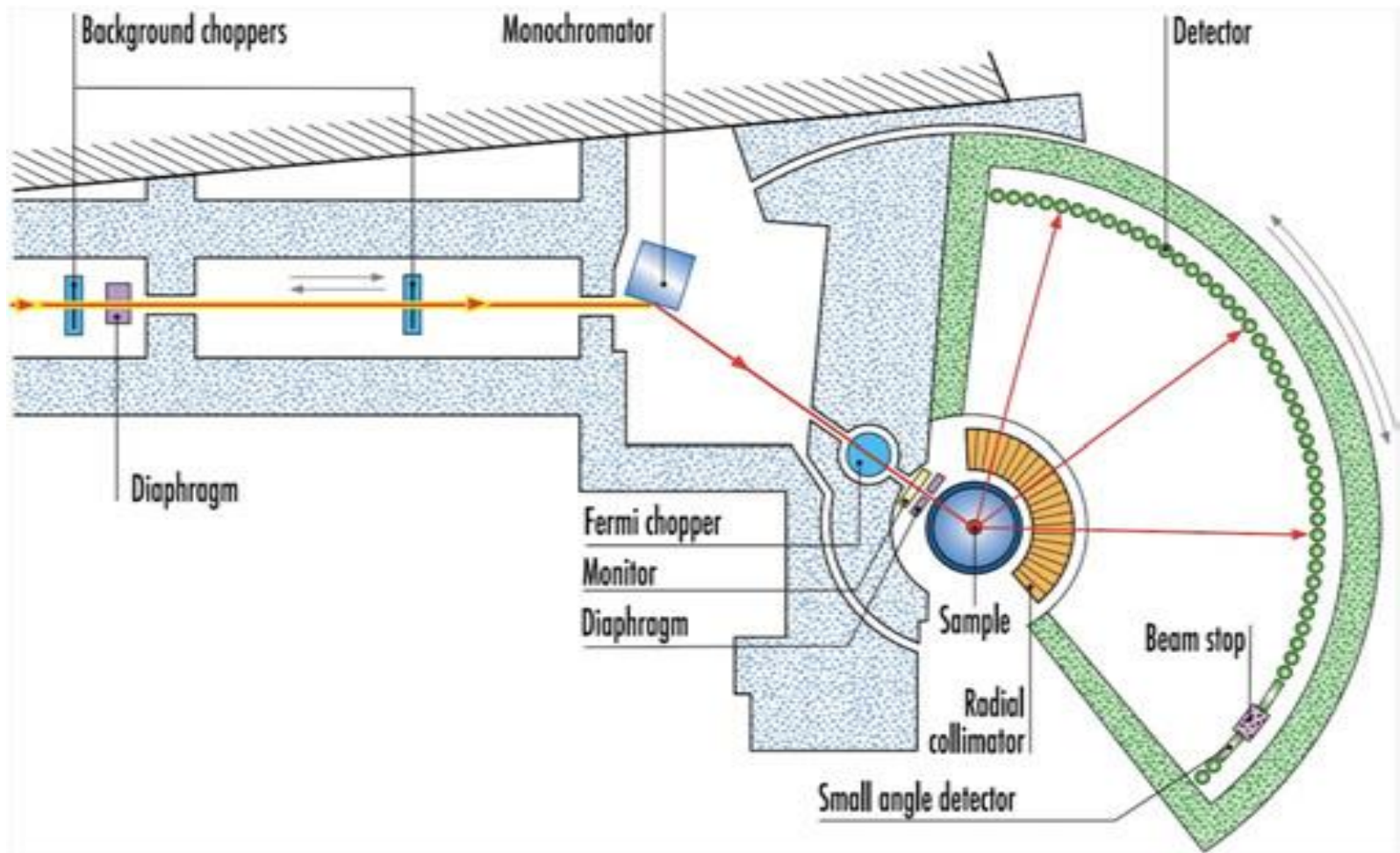
Institut Laue-Langevin

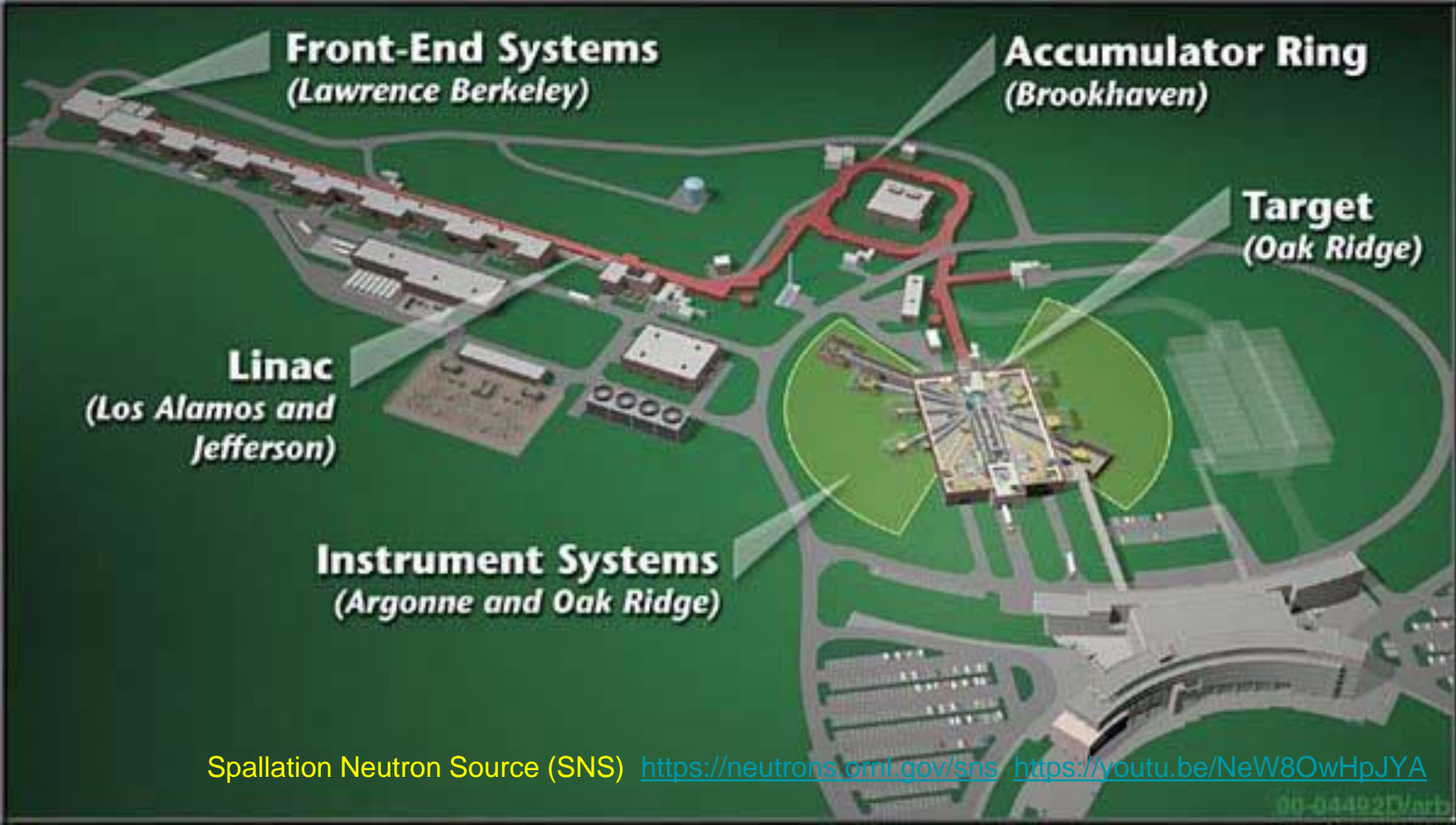
Institut Laue-Langevin <https://youtu.be/xrQgURmcAlc>





- DOUBLE-WALLED REACTOR BUILDING 1
- LEVEL D - REACTOR HALL 2
- CRANE FOR REACTOR OPERATIONS LEVEL D 3
- GANTRY FOR HANDLING OF FUEL ELEMENTS 4
- HOT CELL 5
- LEVEL C - EXPERIMENTAL HALL 6
- CRANE FOR EXPERIMENTAL OPERATIONS 7
- BIOLOGICAL SHIELDING (CONCRETE) 8
- COLLIMATED NEUTRON EXIT POINT 9
- LEVEL B - REACTOR AUXILIARY EQUIPMENT 10
- REACTOR POOL (LIGHT WATER) 11
- HEAVY WATER (MODERATOR & FUEL ELEMENT COOLING) 12
- FUEL ELEMENT 13
- SPENT FUEL ELEMENTS STORAGE 14
- HEAT EXCHANGERS (PRIMARY/SECONDARY) 15
- SECONDARY COOLING CIRCUIT 'DRAC RIVER' 16





  
**Oak Ridge,  
Tennessee**  
LOCATION

  
**2006**  
START OF OPERATIONS

  
**759 (FY 2019)**  
NUMBER OF USERS



**Spallation Neutron Source (SNS)**



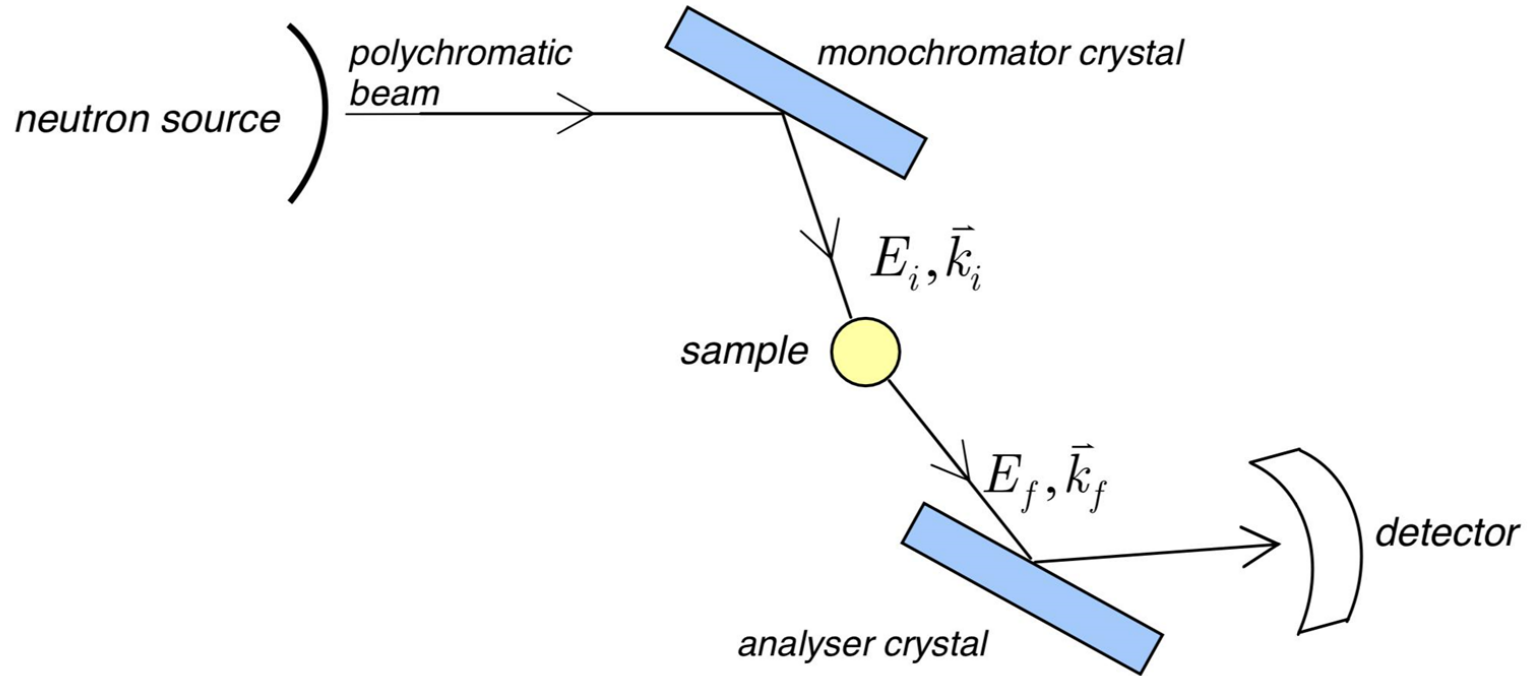
EUROPEAN  
SPALLATION  
SOURCE

<https://europeanspallationsource.se/webcams>

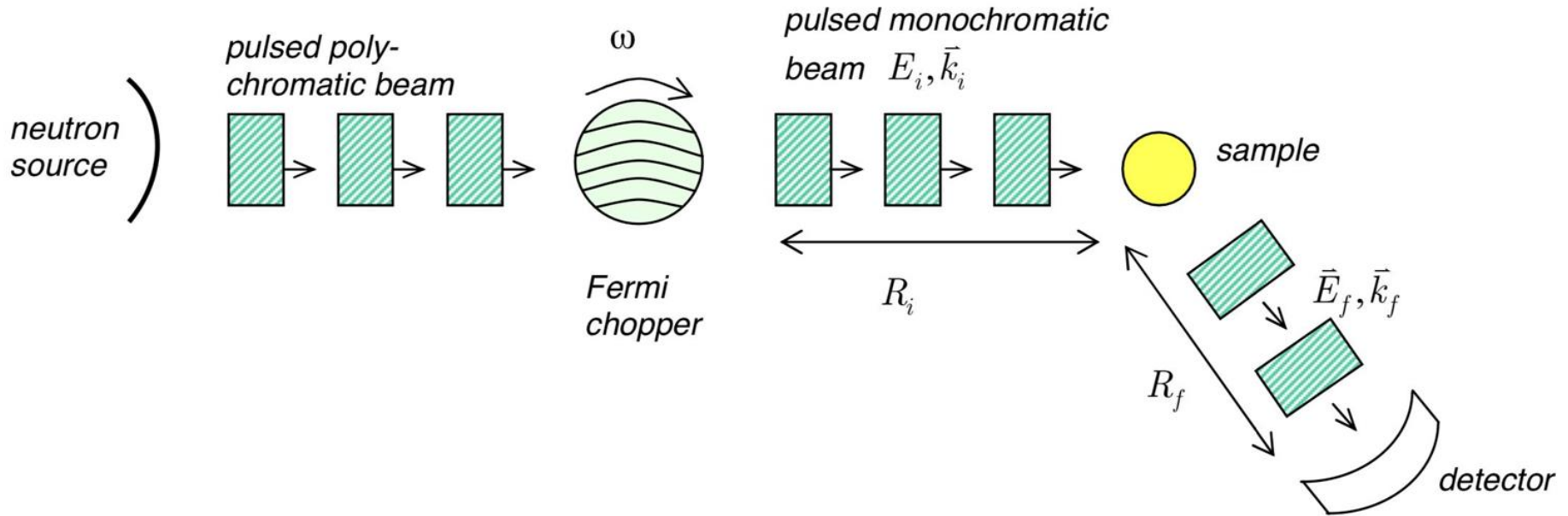


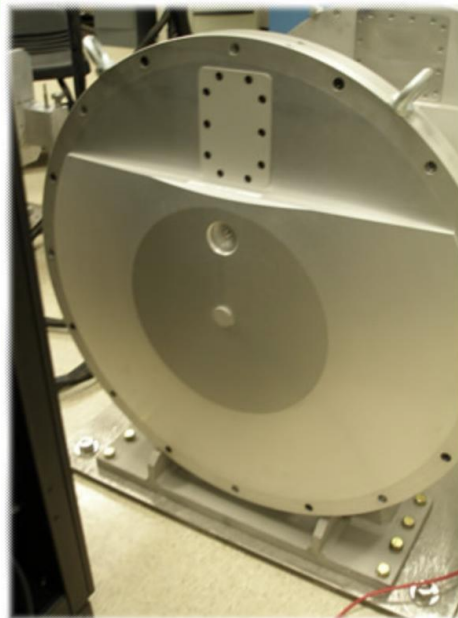
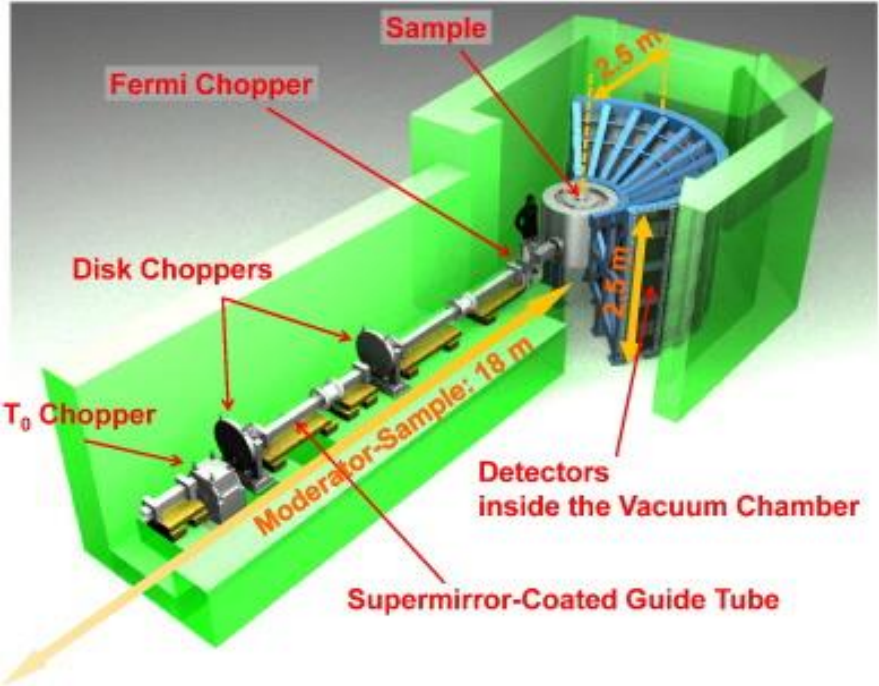


1 — Triple-axis spectrometer  
(commonly used at steady-state neutron sources)

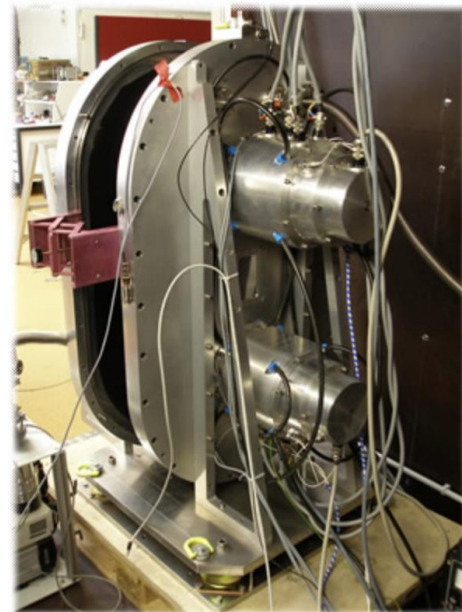


## 2 — Time-of-flight spectrometer (commonly used at pulsed neutron sources)

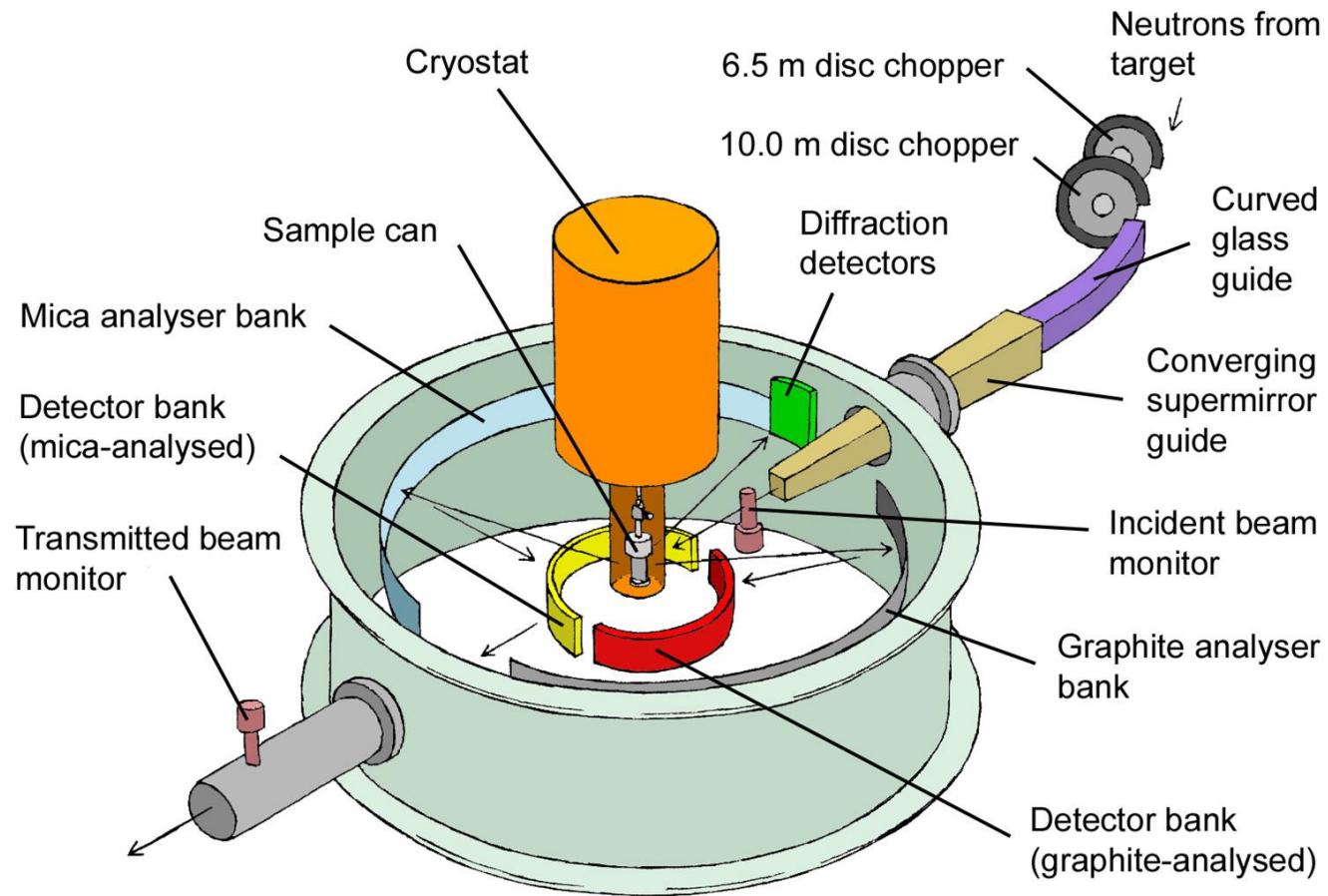




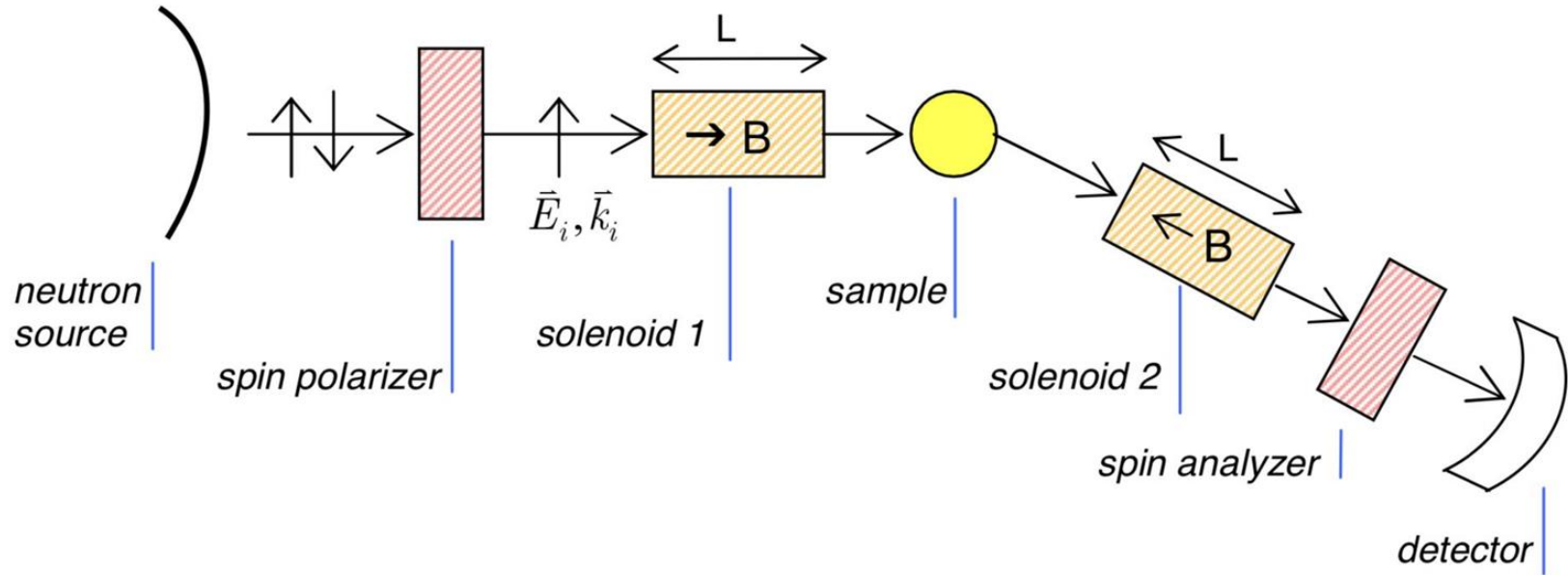
Frame overlap chopper



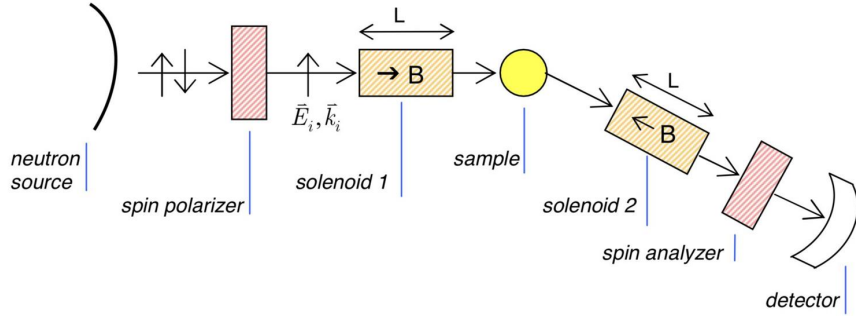
High-speed double disk chopper



### 3 — Spin-echo spectrometer



### 3 – Spin-echo spectrometer

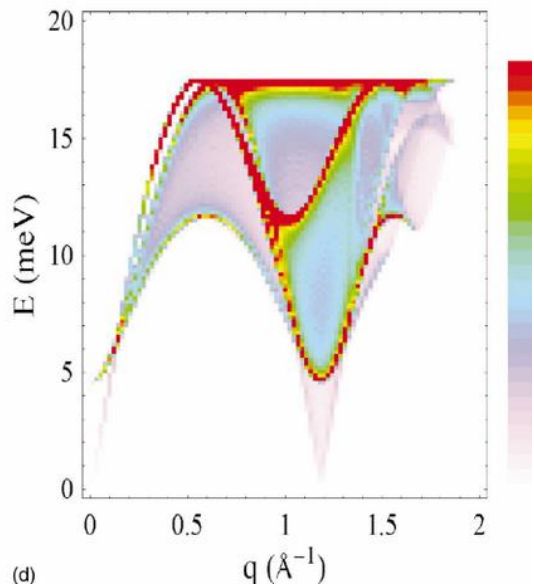
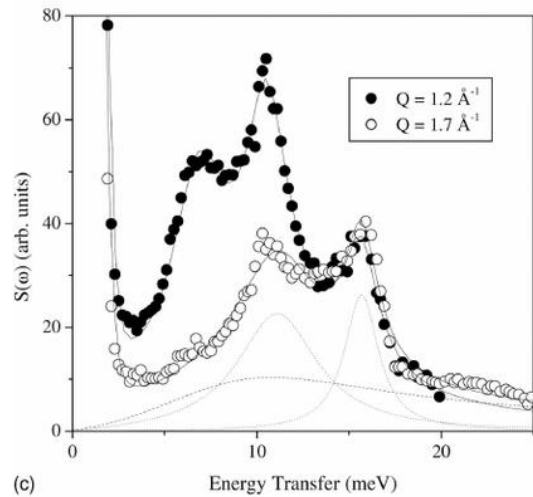
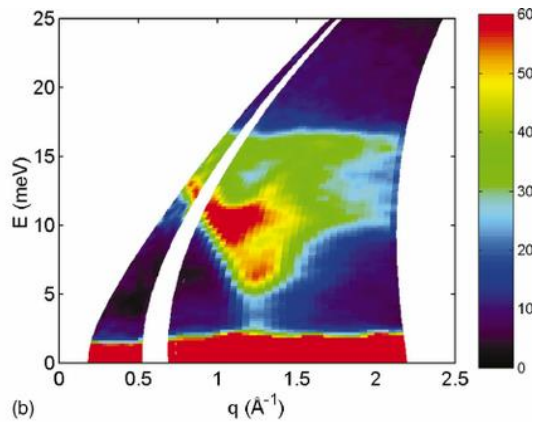
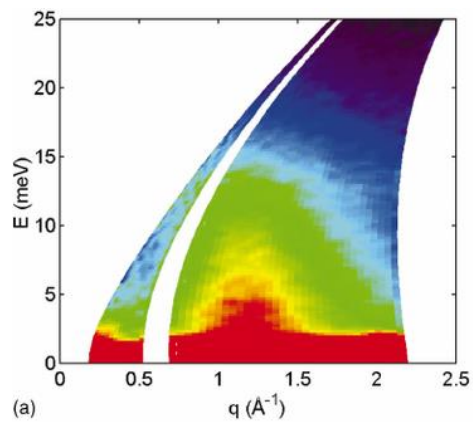


$$\omega_L = \frac{\gamma \mu_N B}{\hbar}$$

$$\phi_i = \frac{\omega_L L}{v_i}$$

$$\Delta\Phi = \Phi_i - \Phi_f = \omega_L L \left( \frac{1}{v_i} - \frac{1}{v_f} \right) = \omega_L L \left( \frac{1}{v_i} - \frac{1}{v_i + \Delta v} \right) \approx \frac{\omega_L L}{v_i^2} \Delta v \quad \text{for } \Delta v \ll v_i$$

$$\hbar\omega = \frac{m_n}{2} (v_f^2 - v_i^2) \approx m_n v_i \Delta v \Rightarrow \Delta\Phi \approx \frac{\hbar\omega_L L}{m_n v_i^3} \omega \equiv \omega \tau_{SE} \quad \tau_{SE} = \text{spin echo time}$$



Magnetic ordering and spin-liquid state of YMnO<sub>3</sub> - Park *PRB* 2003