



*Курс:*

Фізичні методи дослідження матеріалів

*Тема:*

Квантове тунелювання та тунельна спектроскопія:  
FT-STIS

*Лектор:* О. А. Кордюк

# Spectroscopic Techniques

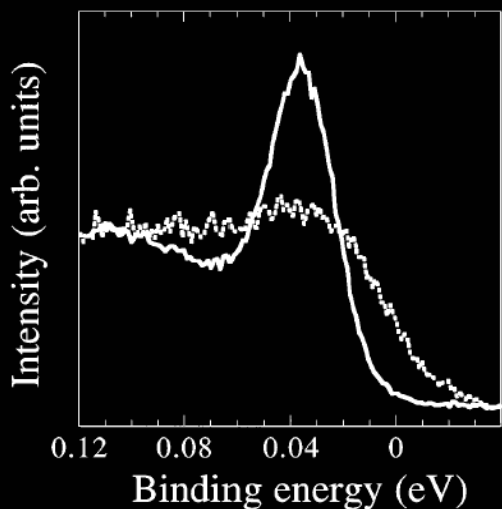
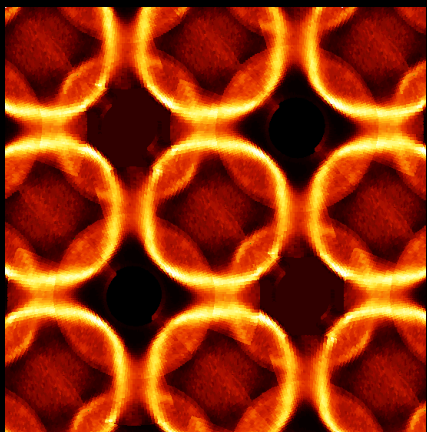
In \ Out	$h\nu$	<i>electrons</i>	<i>A</i>
$h\nu$	<b>XD, IR, Raman</b>	<b>ARPES, XPS...</b>	<b>LA</b>
$e$	<b>IPS, EDX (SEM)</b>	<b>SEM, LEED, EELS</b>	<b>ESD</b>
<i>A</i>	<b>BLE</b>	<b>IAES</b>	<b>RBS, SIMS</b>
<i>T</i>			<b>TDS</b>
<i>E</i>		<b>STM/STS, FEM</b>	<b>FIM</b>

$n-n$ : ND, INS

$\mu-e^+$ :  $\mu$ SR

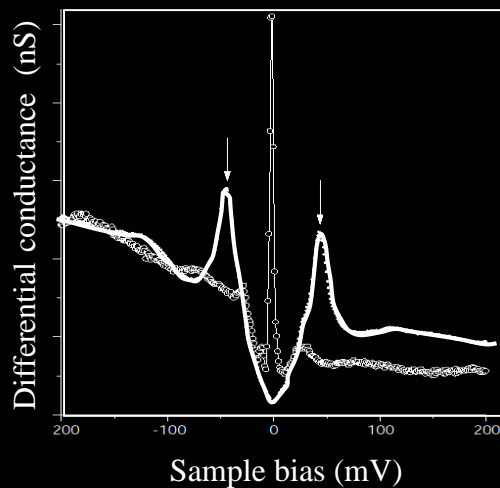
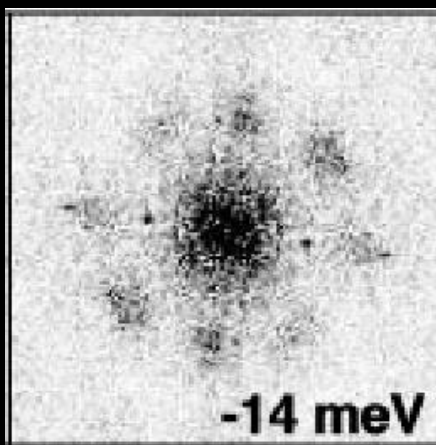
# ARPES

Angle Resolved Photo-emission Spectroscopy



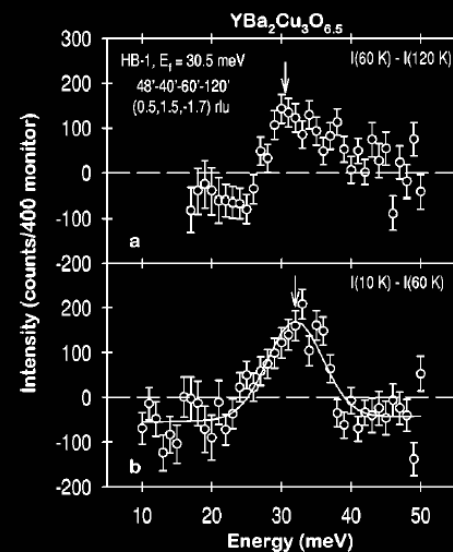
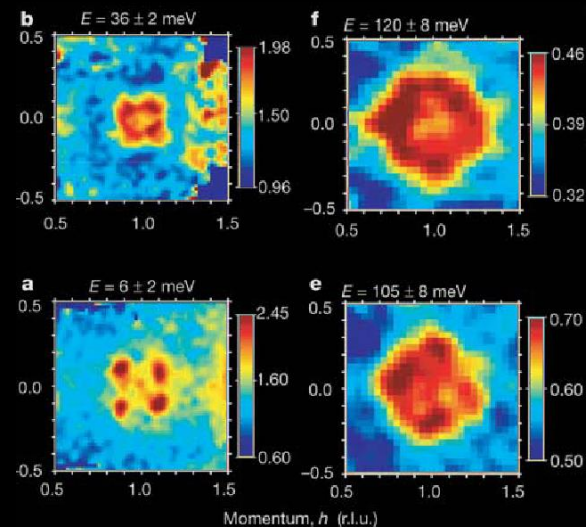
# STS

Scanning Tunneling Spectroscopy

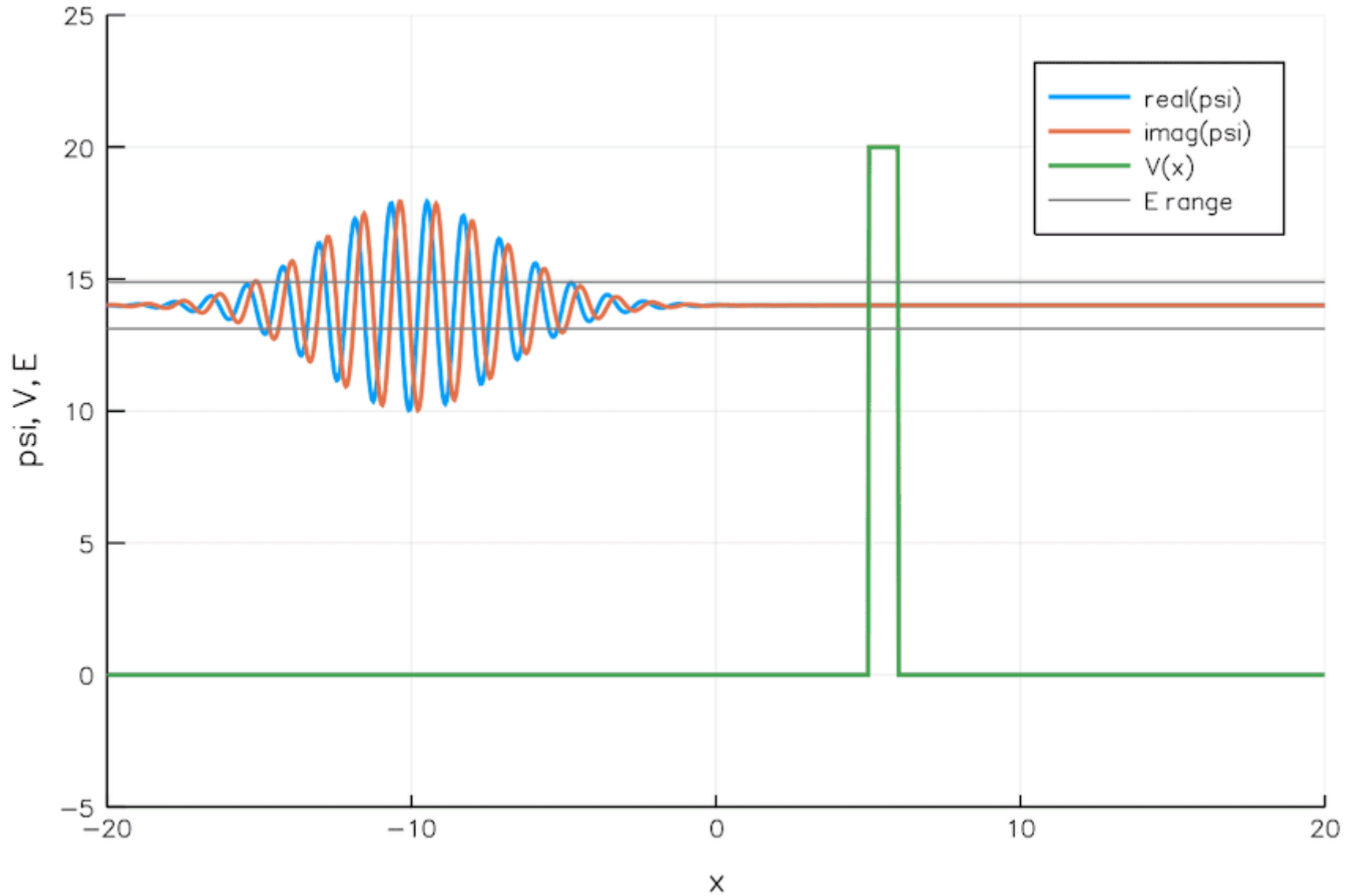


# INS

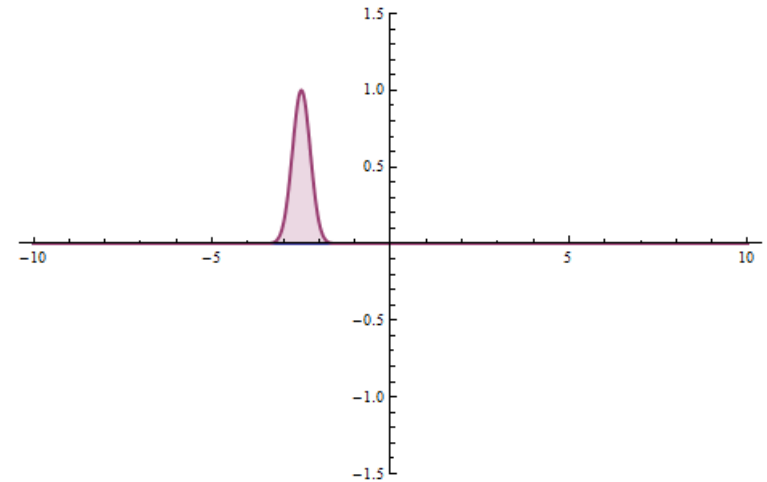
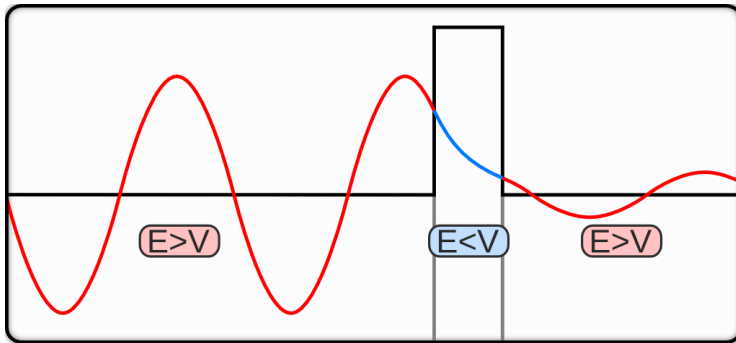
Inelastic Neutron Scattering



# Quantum tunnelling



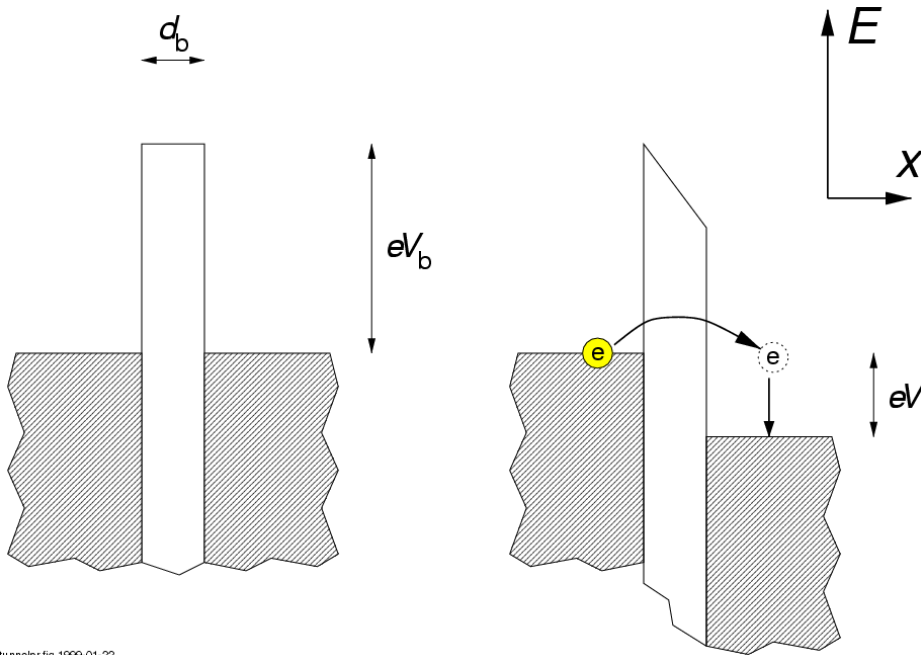
# Quantum tunnelling



$$T(E) = e^{-2 \int_{x_1}^{x_2} dx \sqrt{\frac{2m}{\hbar^2} [V(x) - E]}} = e^{-2 \sqrt{\frac{2m}{\hbar^2} (V_0 - E)(x_2 - x_1)}}$$

- Nuclear fusion
- Tunnel diode
- Scanning tunneling microscopy (STM)
- Quantum computing

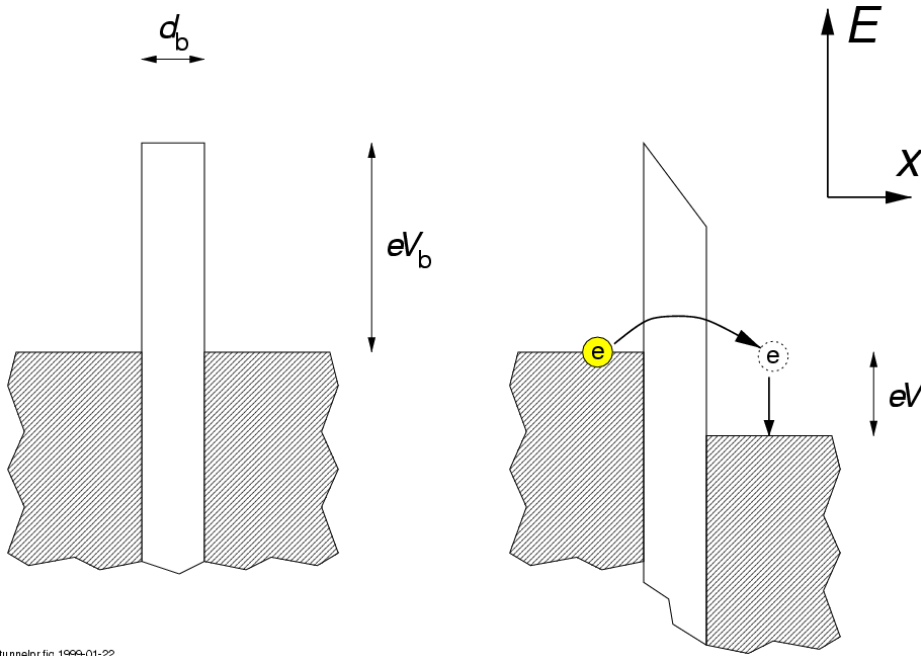
# Tunnel junction



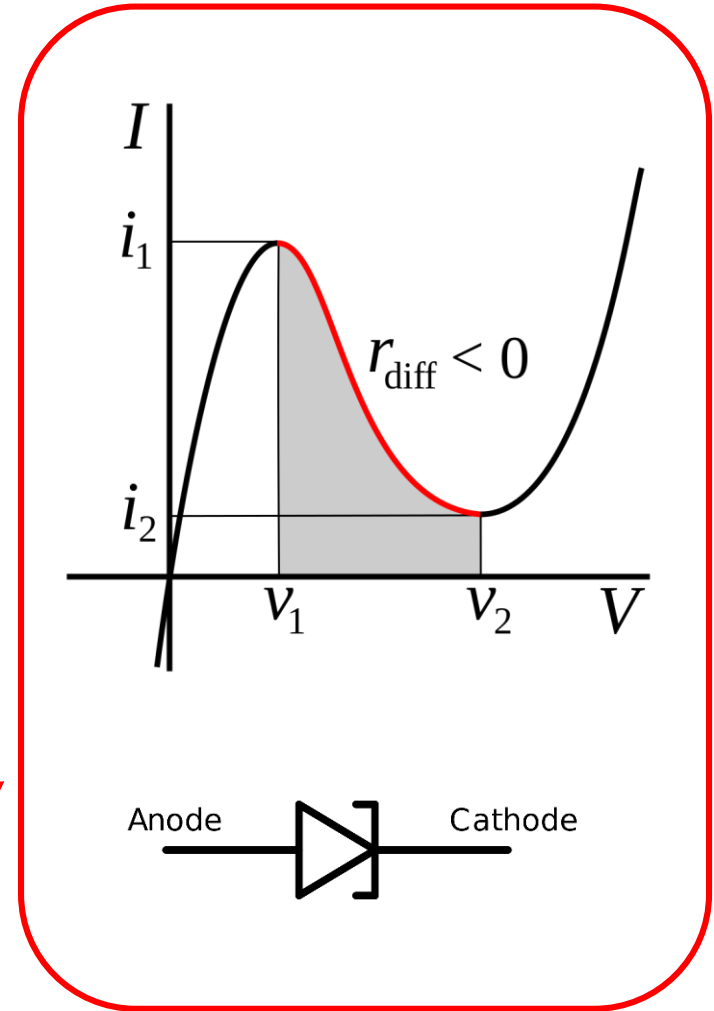
tunnelpr.fig 1999-01-22

- Multijunction photovoltaic cell
- Tunnel diode
- Magnetic tunnel junction
- Superconducting tunnel junction
- Scanning tunneling microscope

# Tunnel junction

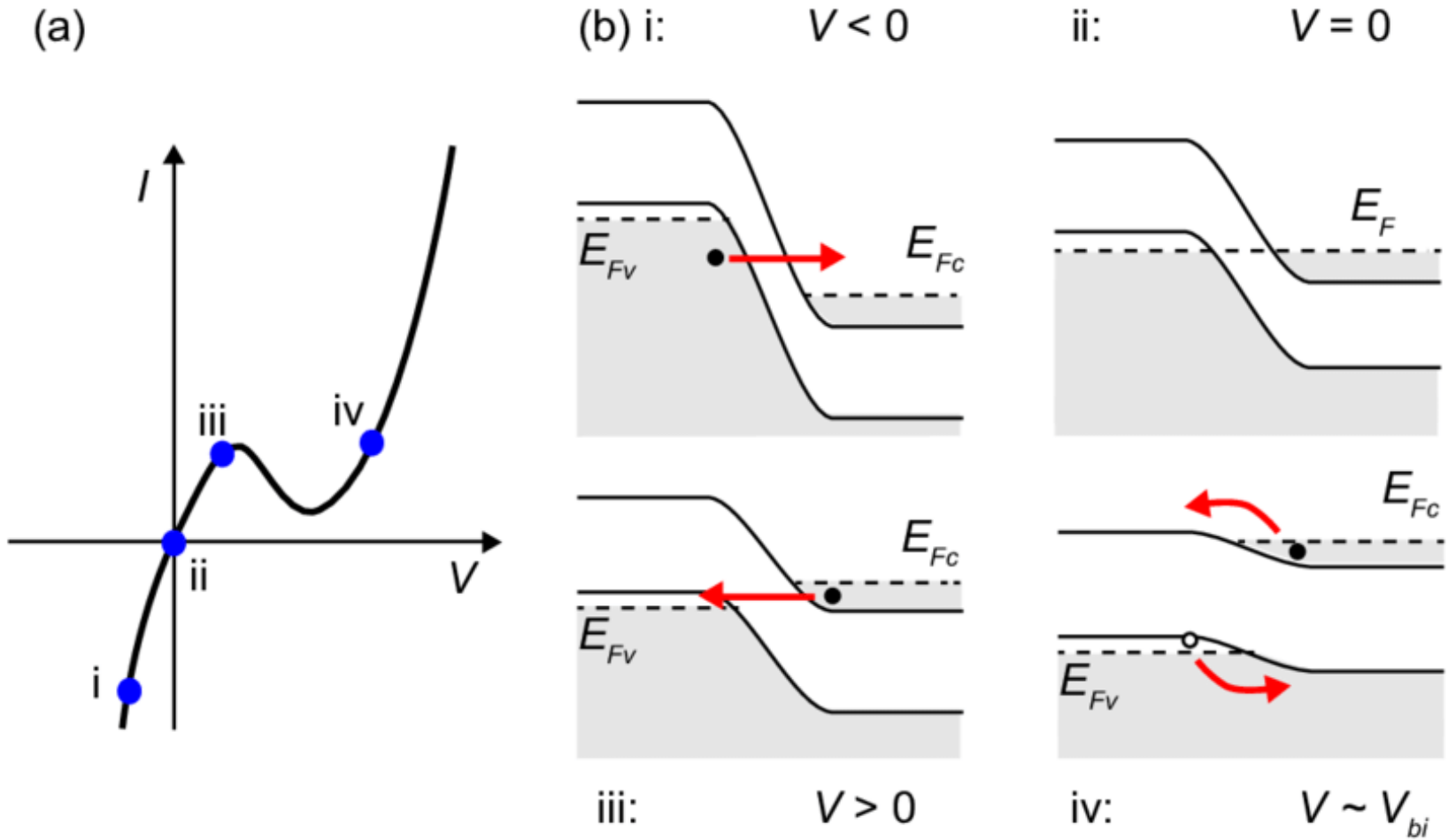


tunnelpr.fig 1999-01-22



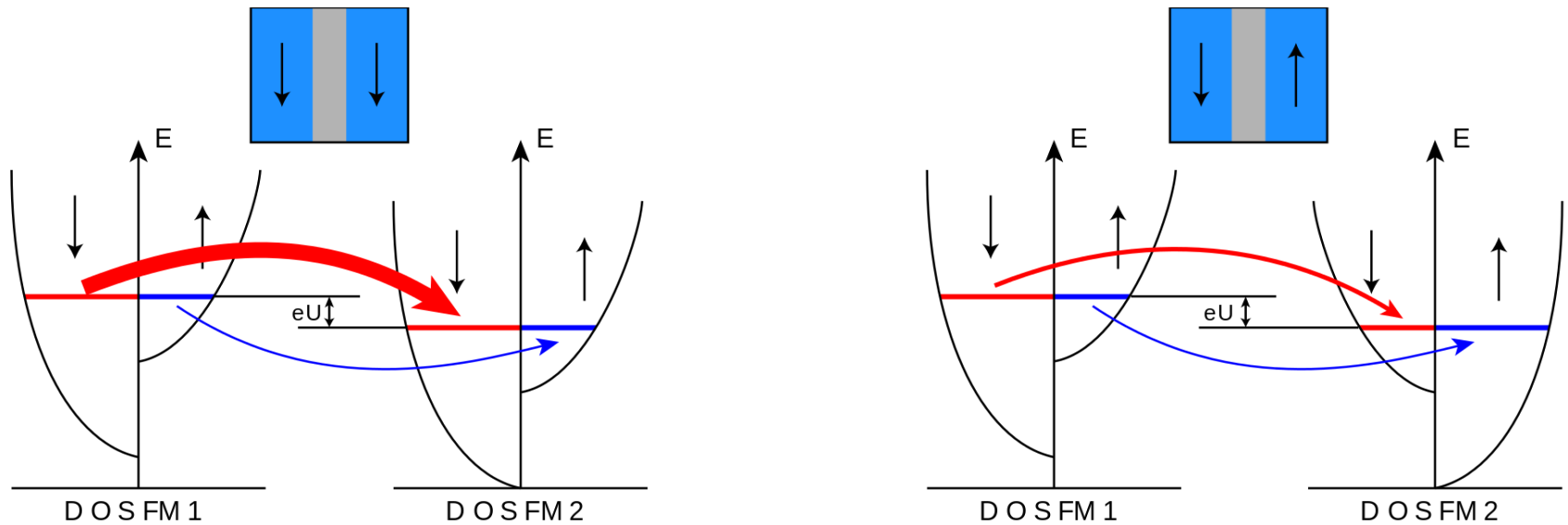
- Multijunction photovoltaic cell
- **Tunnel diode**
- Magnetic tunnel junction
- Superconducting tunnel junction
- Scanning tunneling microscope

# Tunnel diode





# Tunnel magnetoresistance



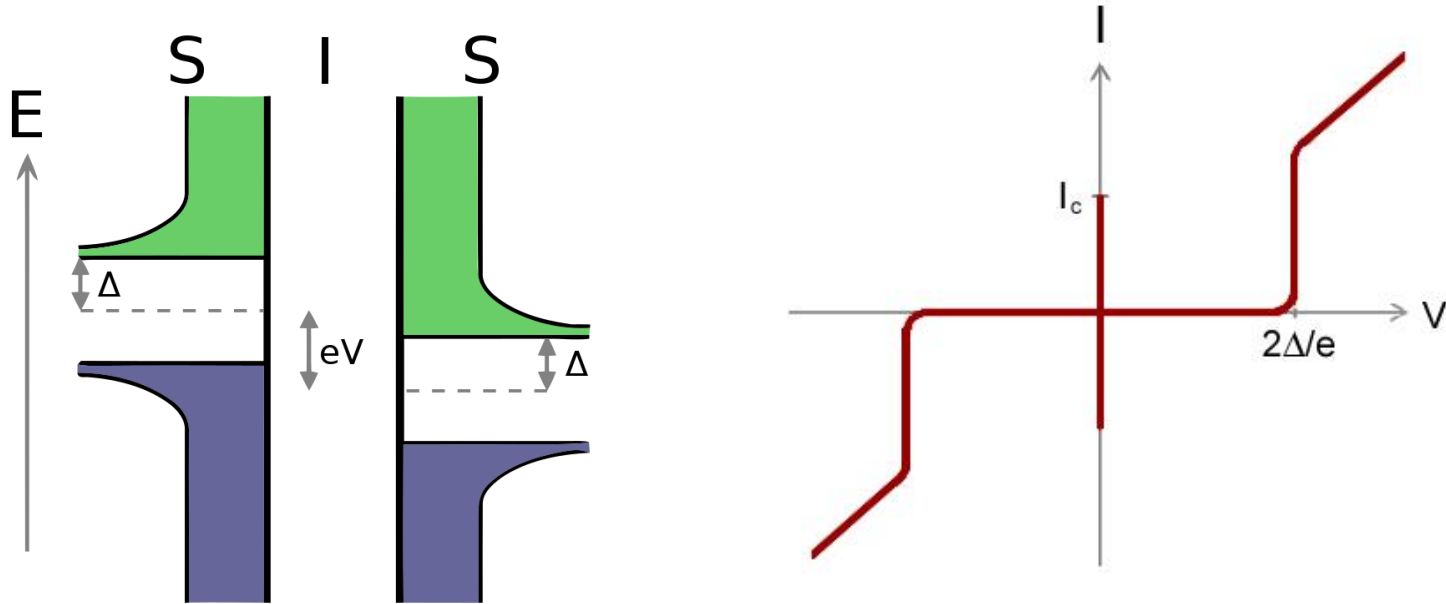
$$\text{TMR} := \frac{R_{\text{ap}} - R_{\text{p}}}{R_{\text{p}}}$$

$$P = \frac{\mathcal{D}_{\uparrow}(E_{\text{F}}) - \mathcal{D}_{\downarrow}(E_{\text{F}})}{\mathcal{D}_{\uparrow}(E_{\text{F}}) + \mathcal{D}_{\downarrow}(E_{\text{F}})}$$

$$\text{TMR} = \frac{2P_1 P_2}{1 - P_1 P_2}$$

where  $R_{\text{ap}}$  is the electrical resistance in the anti-parallel state, whereas  $R_{\text{p}}$  is the resistance in the parallel state.  $P$  is calculated from the spin dependent density of states (DOS)  $\mathcal{D}$  at the Fermi energy

# Superconducting tunnel junction



## *Applications*

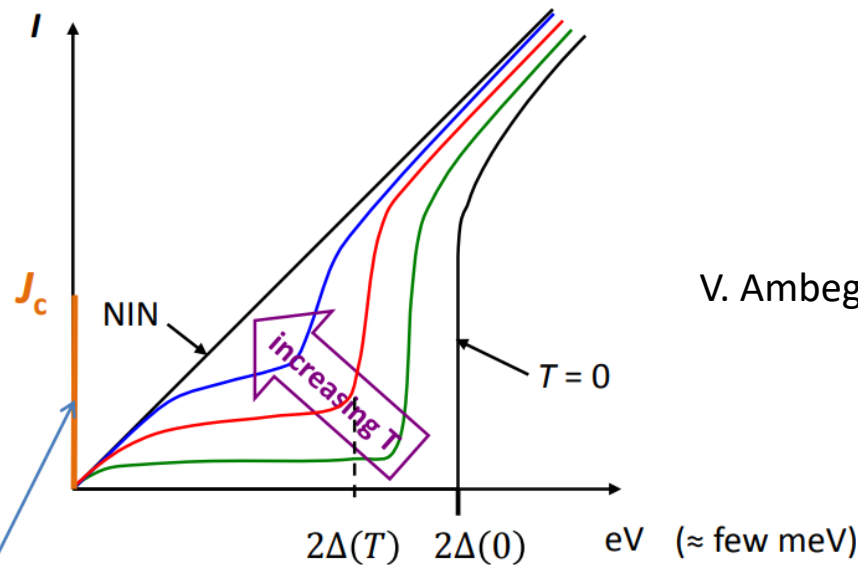
- **Radio astronomy:** photon-assisted tunneling -> the most sensitive heterodyne
- **Single-photon detection:** photon breaks Cooper pairs creating quasiparticles
- **SQUIDs**
- Superconducting quantum computing
- RSFQ
- Josephson voltage standard

# Superconducting tunnel junction

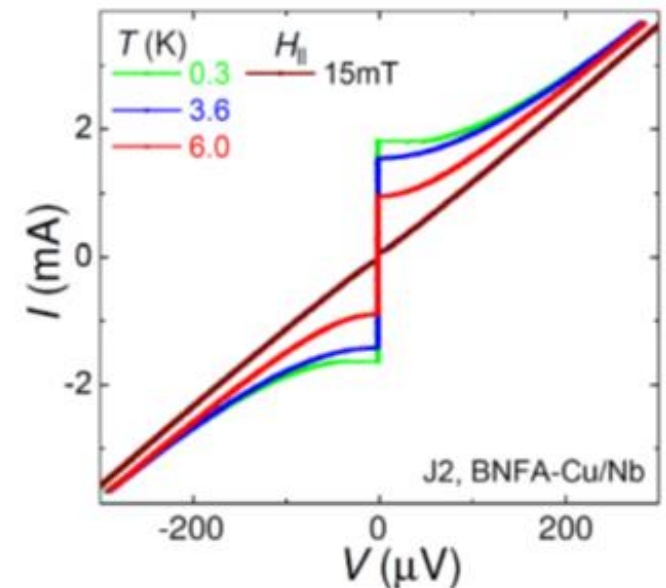
Ambegaokar-Baratoff relation:

$$I_c R_n = \frac{\pi}{2e} \Delta(T) \cdot \tanh \left( \frac{\Delta(T)}{2k_B T} \right)$$

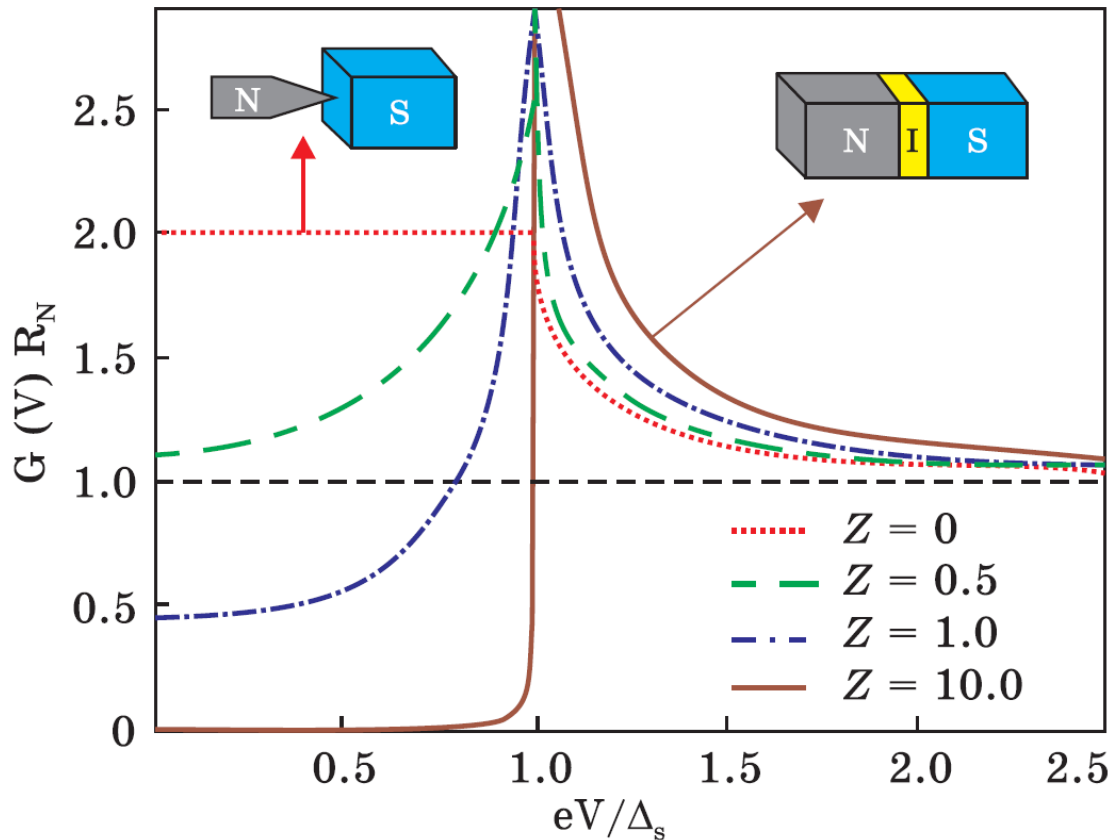
V. Ambegaokar, A. Baratoff, *Phys. Rev. Lett.* **10**, 486 (1963)



• Cooper pair tunneling: 
$$J_c = \frac{e\hbar\kappa}{m} 2\sqrt{n_1 n_2} \exp(-2\kappa d)$$



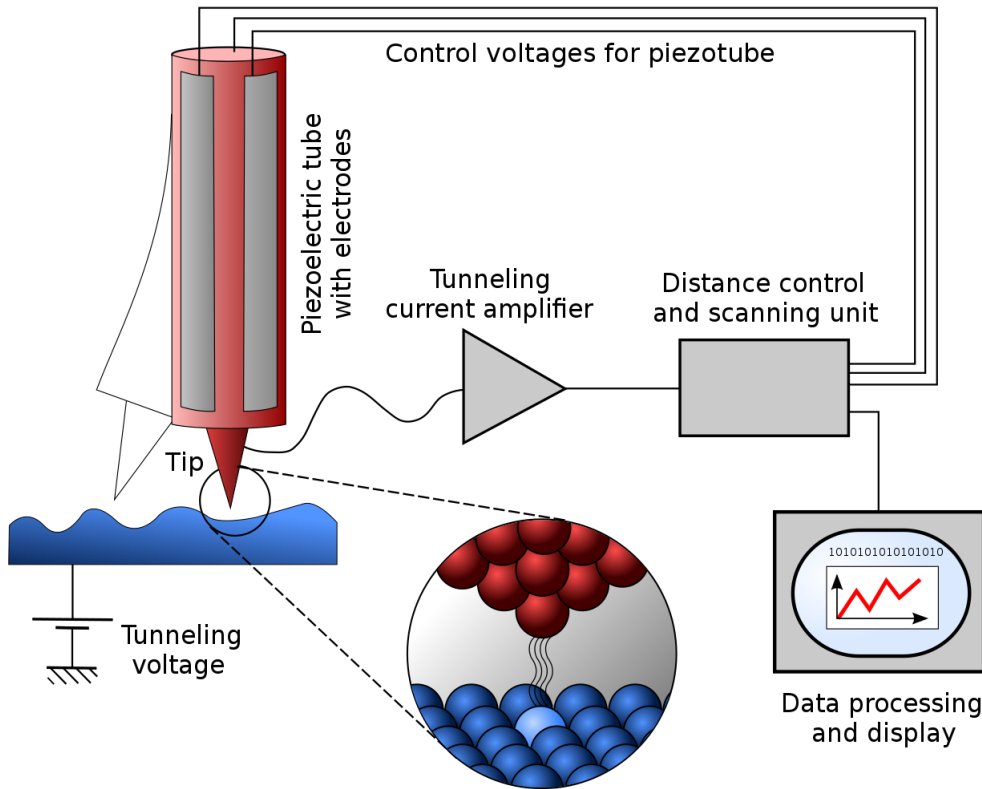
# Superconducting tunnel junction



Differential conductance vs voltage for zero-temperature coherent quantum transport across a one-dimensional N/B/S-trilayer with various barrier transparencies [S. Volkov et al., Appl. Nanosci. (2021)]

$Z$  determines probability of electron transmission  $D = 1/(1 + Z^2)$   
and reflection  $R = 1 - D = Z^2/(1 + Z^2)$

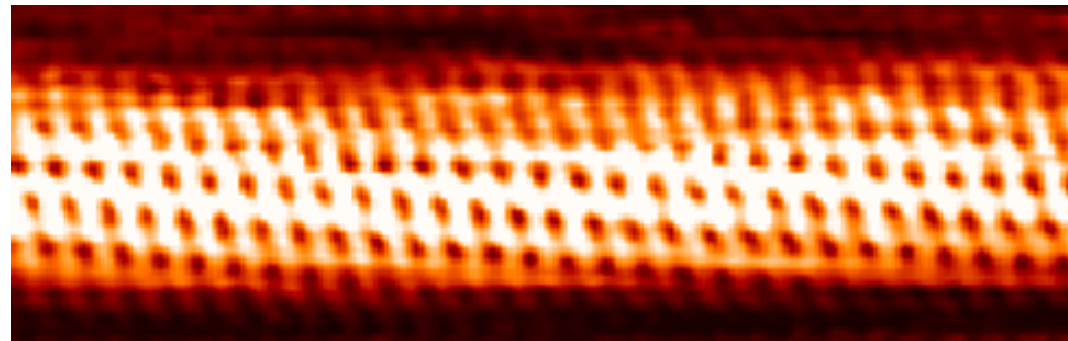
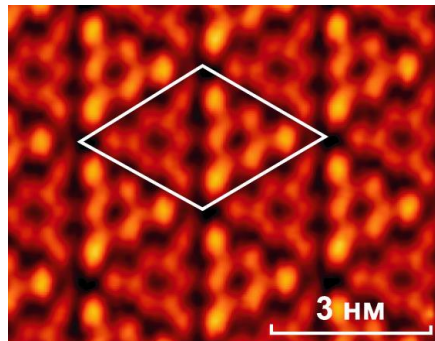
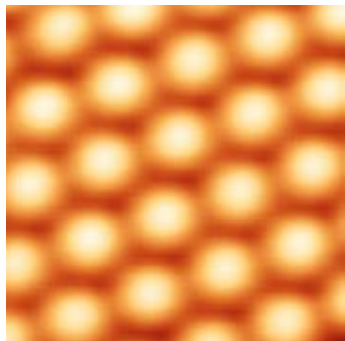
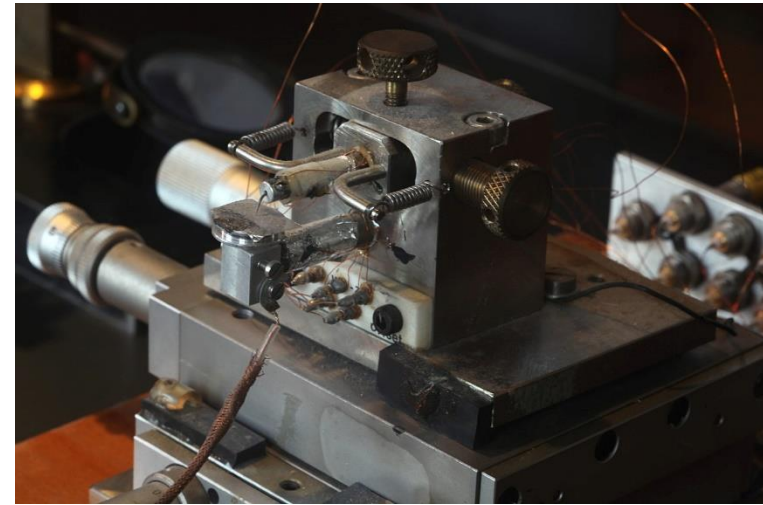
# Scanning tunneling microscope (STM)



**1981**

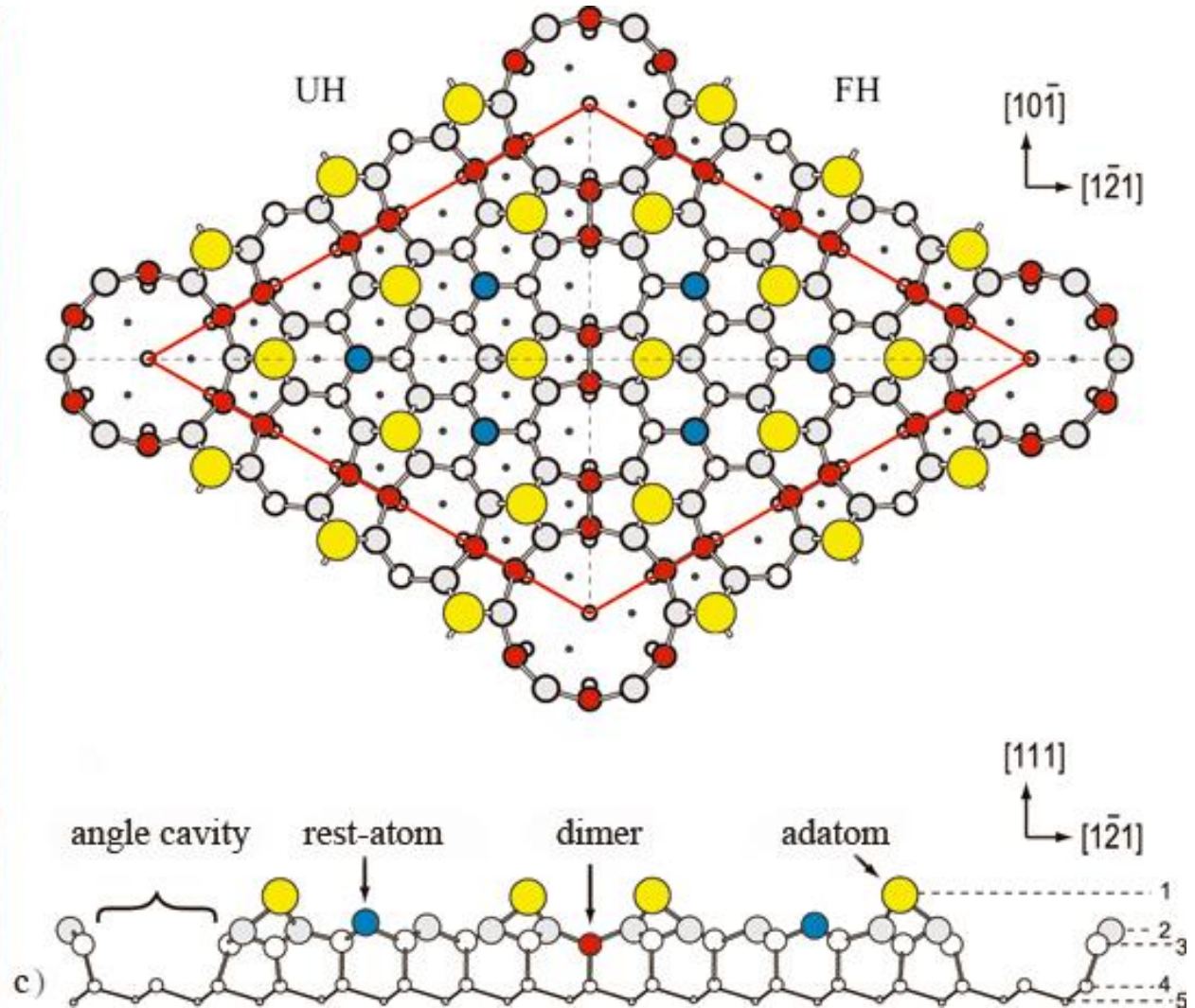
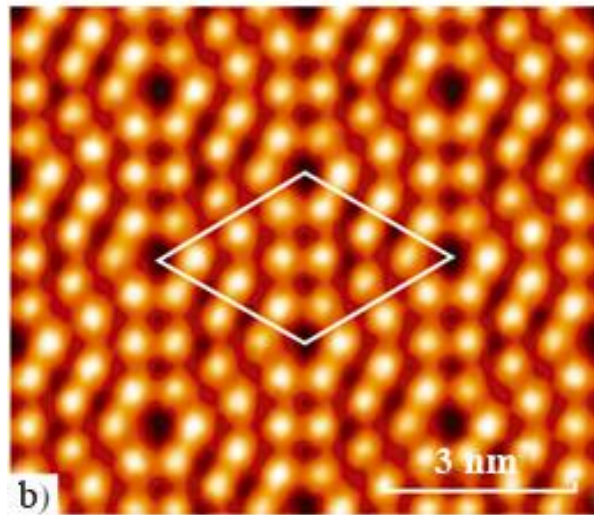
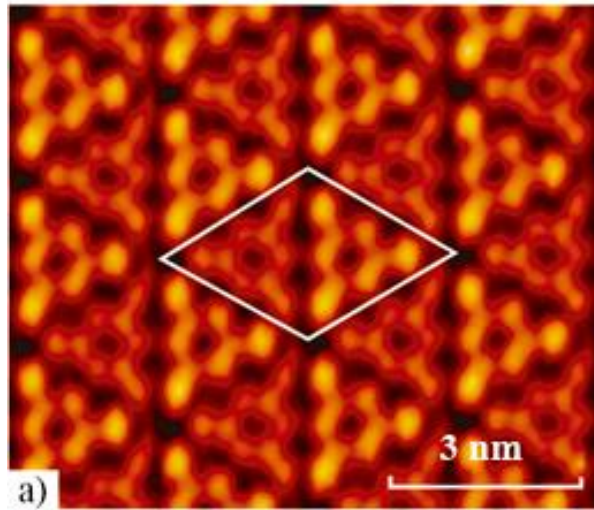
Gerd Binnig and Heinrich Rohrer  
(IBM Zürich),

Nobel Prize in Physics in 1986

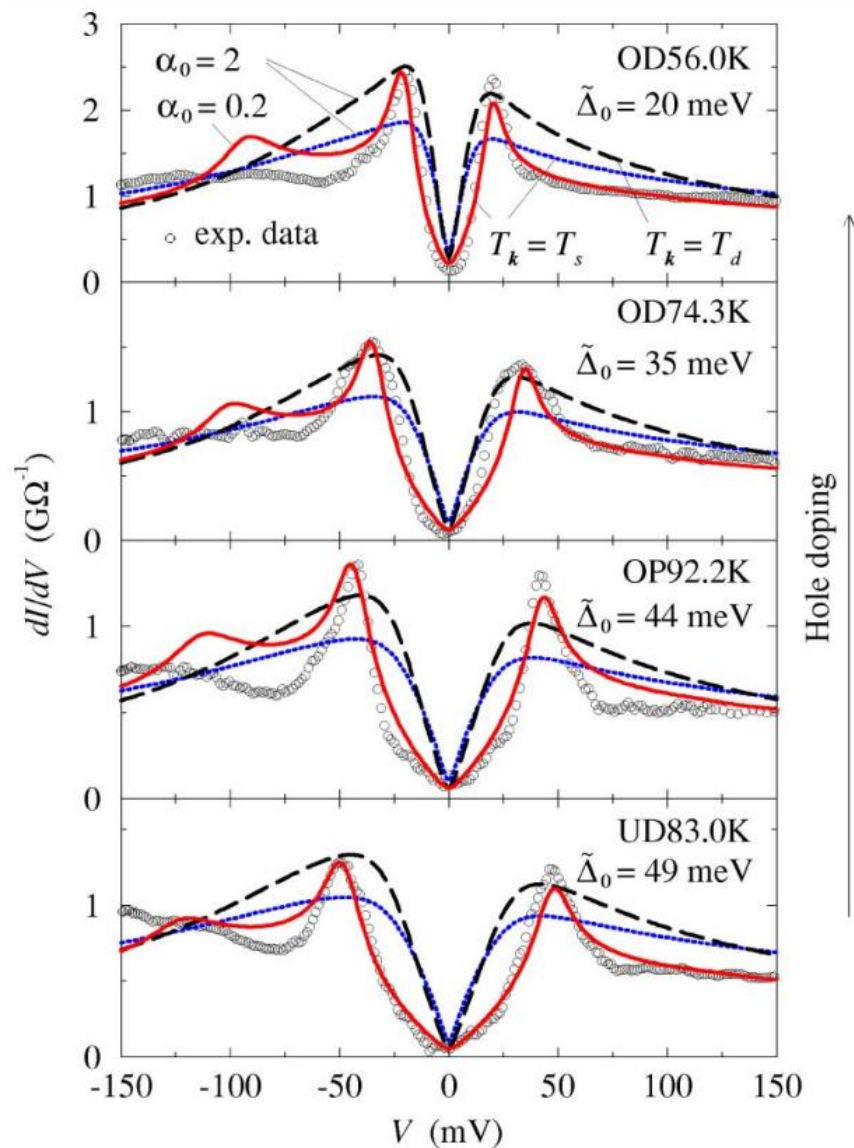
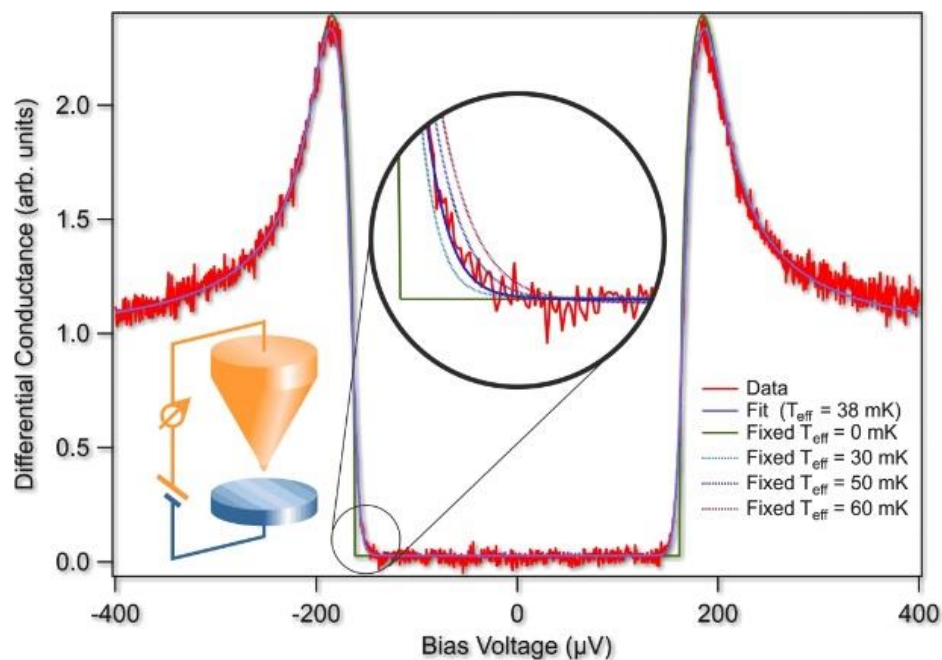
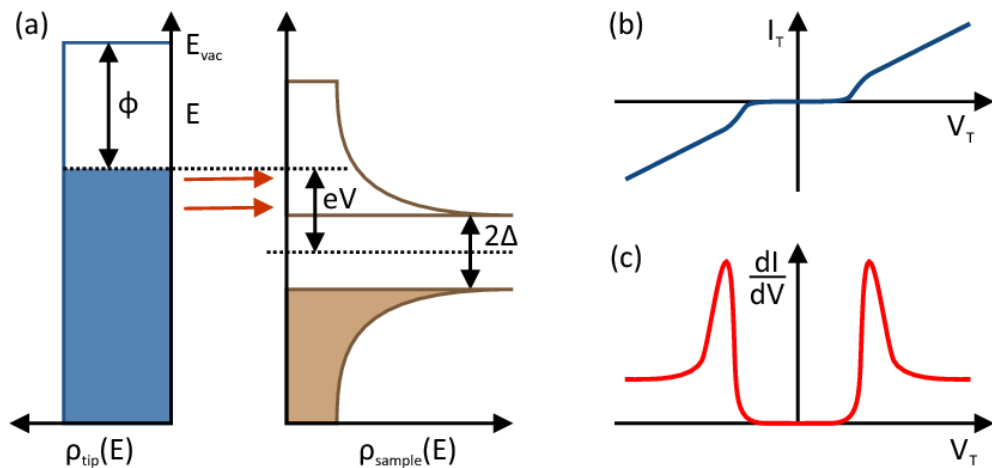




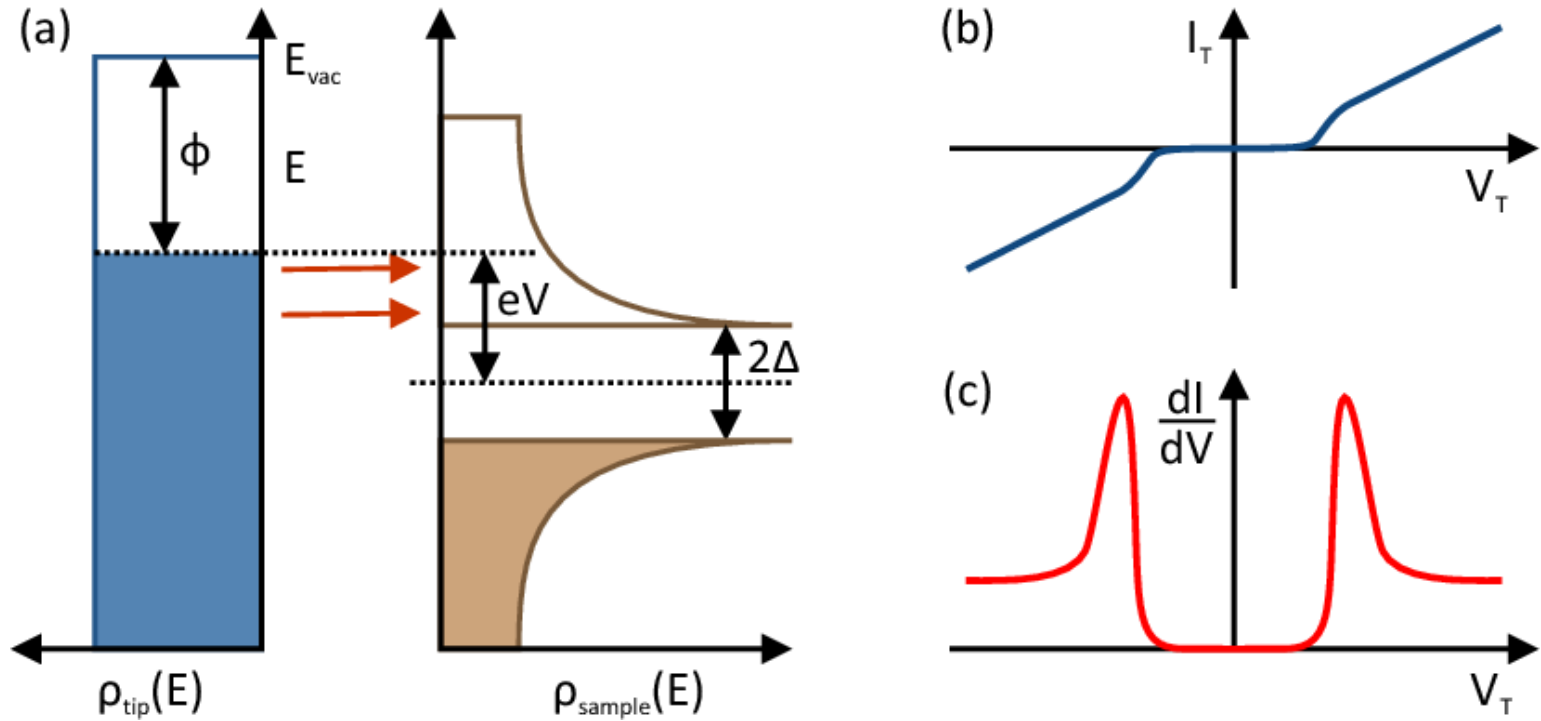
# Si(111) 7x7 superstructure



# Scanning tunneling spectroscopy (STS)



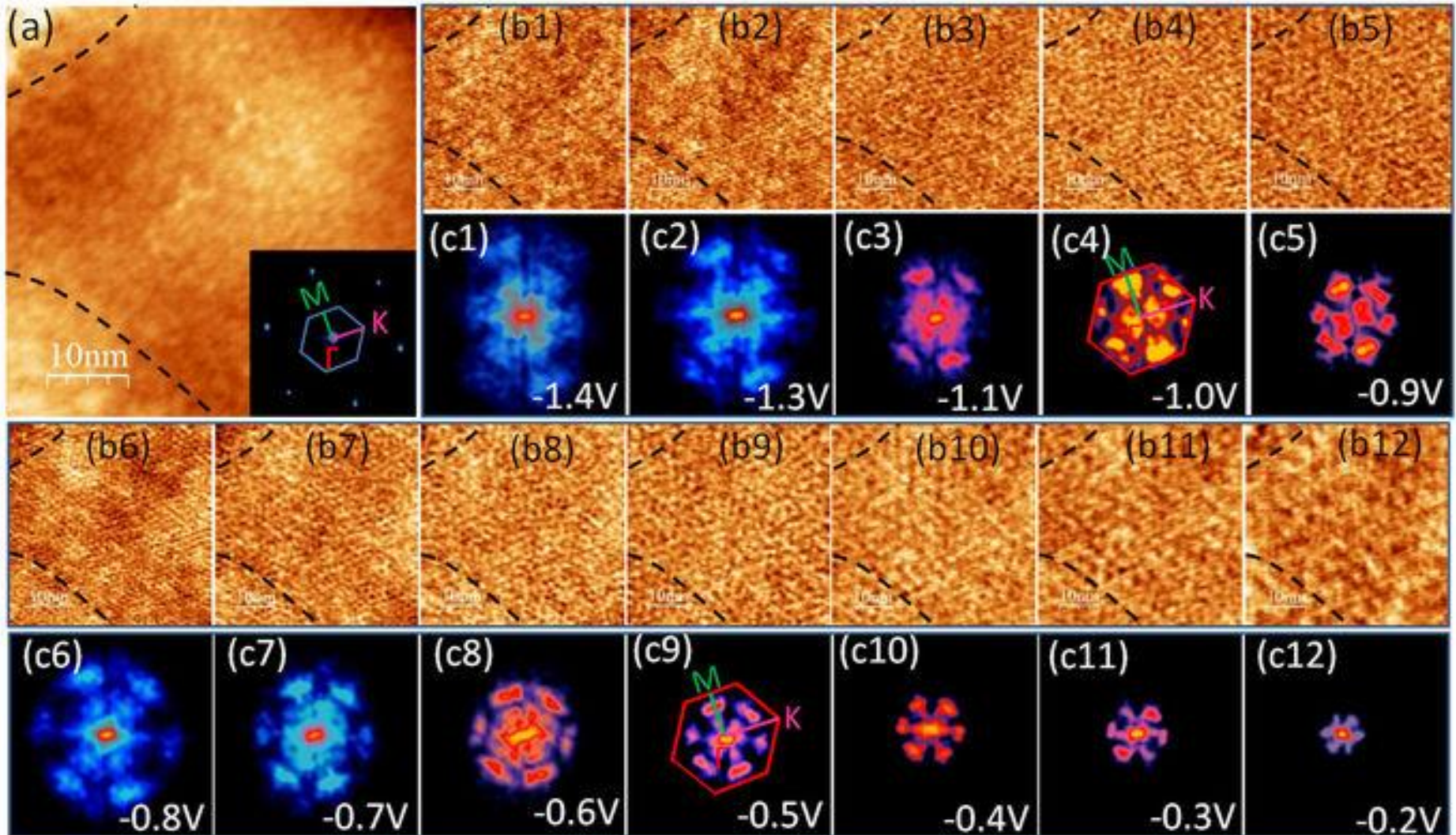
# Scanning tunneling spectroscopy (STS)



$$\frac{dI}{dV} \propto - \int d\omega \sum_{\mathbf{k}, n} |T_{\mathbf{k}}|^2 A_n(\mathbf{k}, \omega) f'(\omega - eV)$$

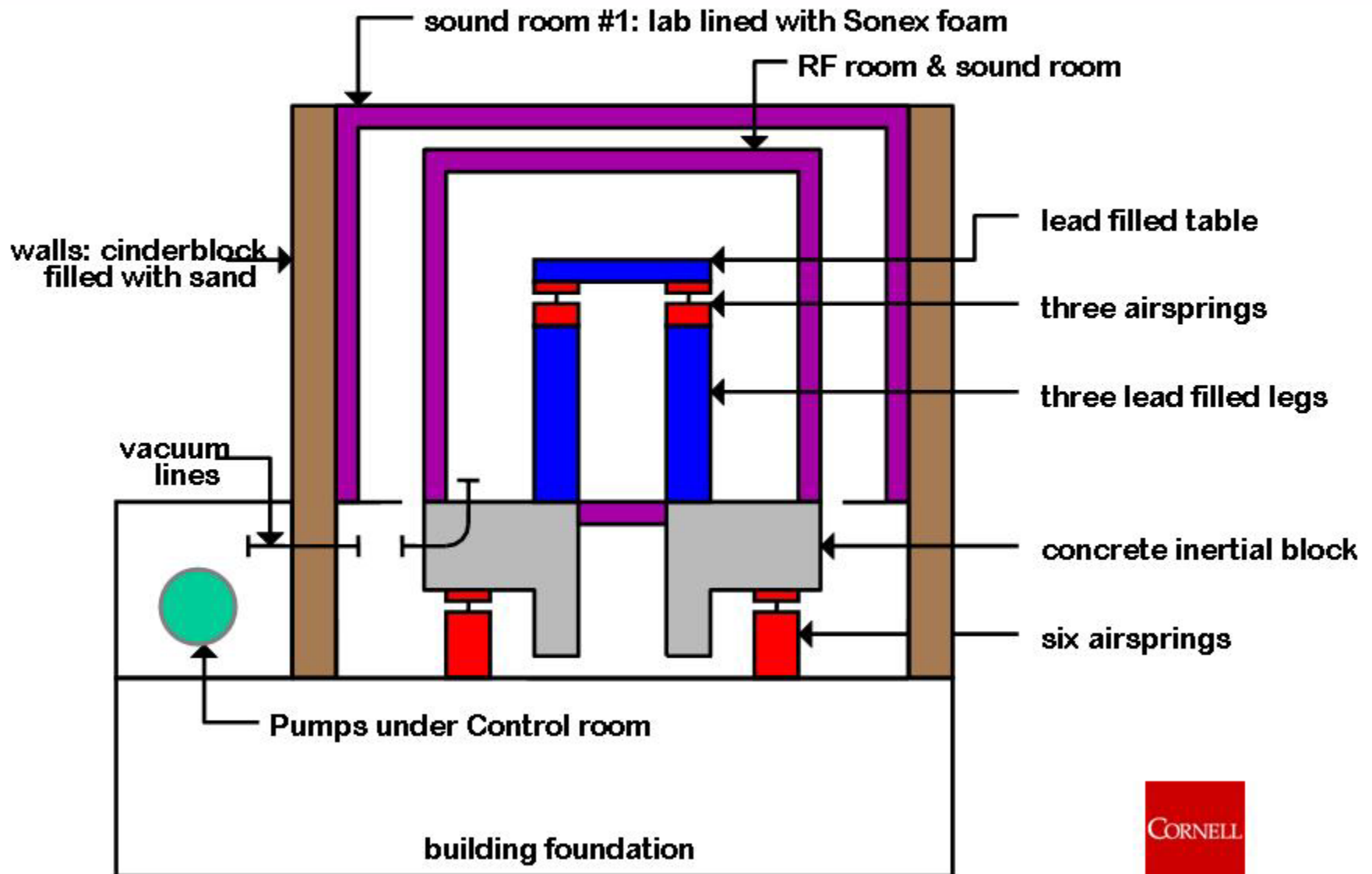


# Fourier Transform STS



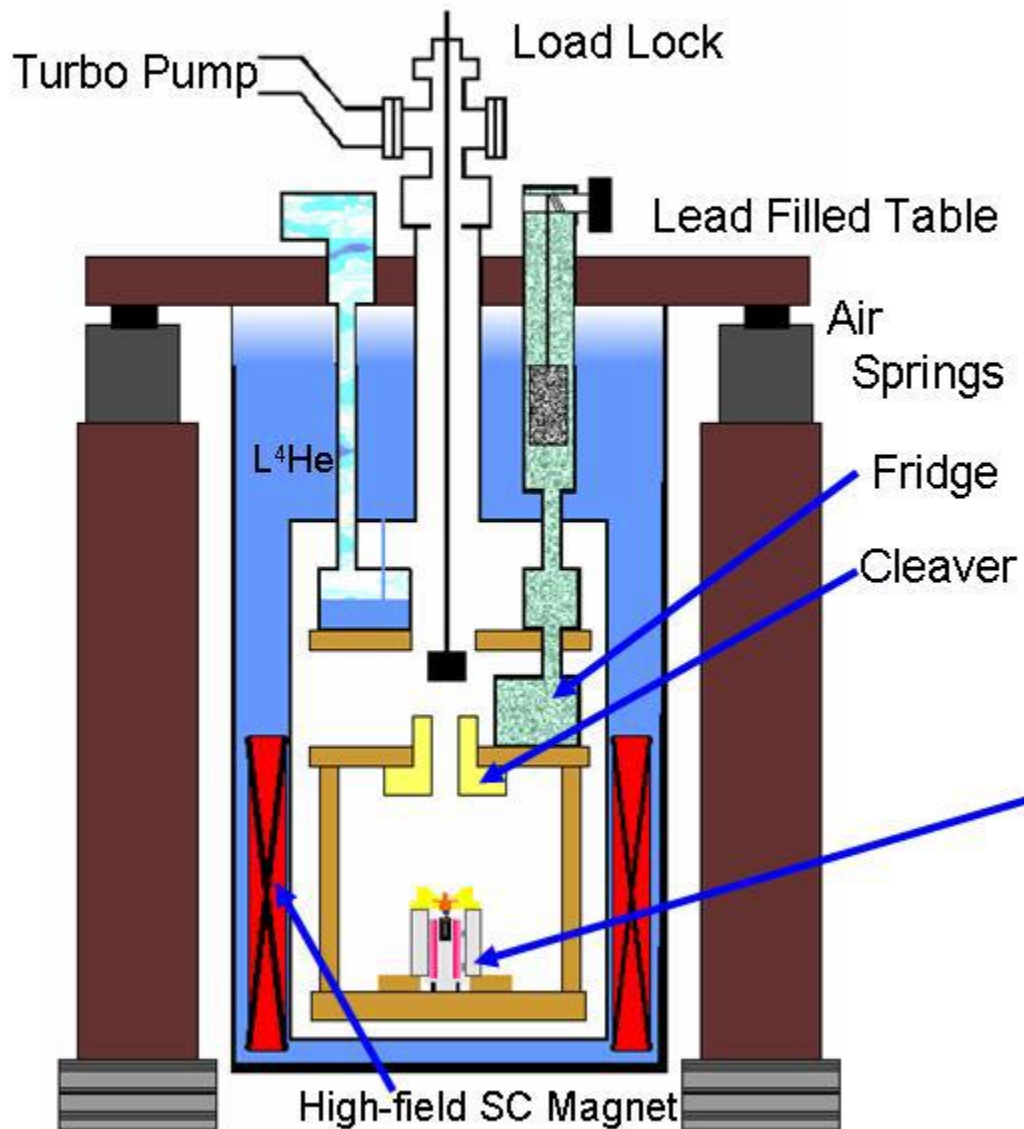
# Floating room

McElroy, 2004

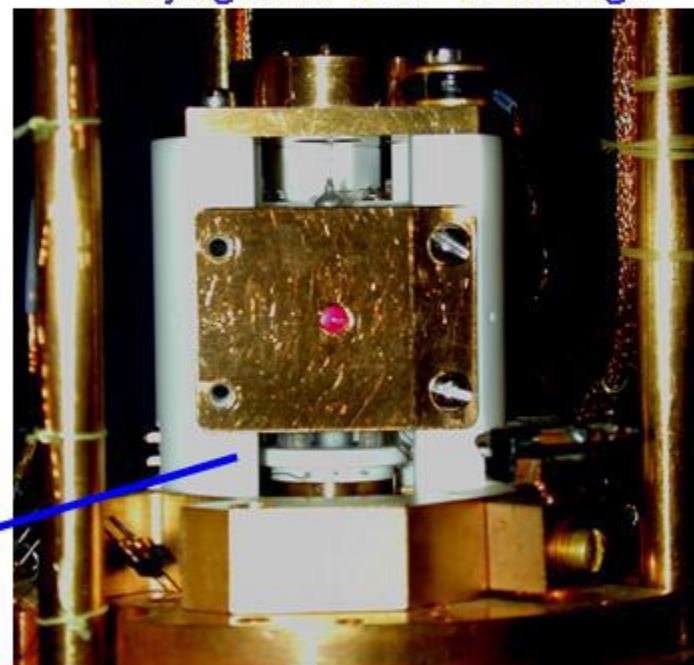


# STM Instrument Design

McElroy, 2004



- 'Nuclear Demag.' Cryostat
- STM + High Field Magnet
- Sample Exchange from RT
- Cryogenic UHV Cleavage



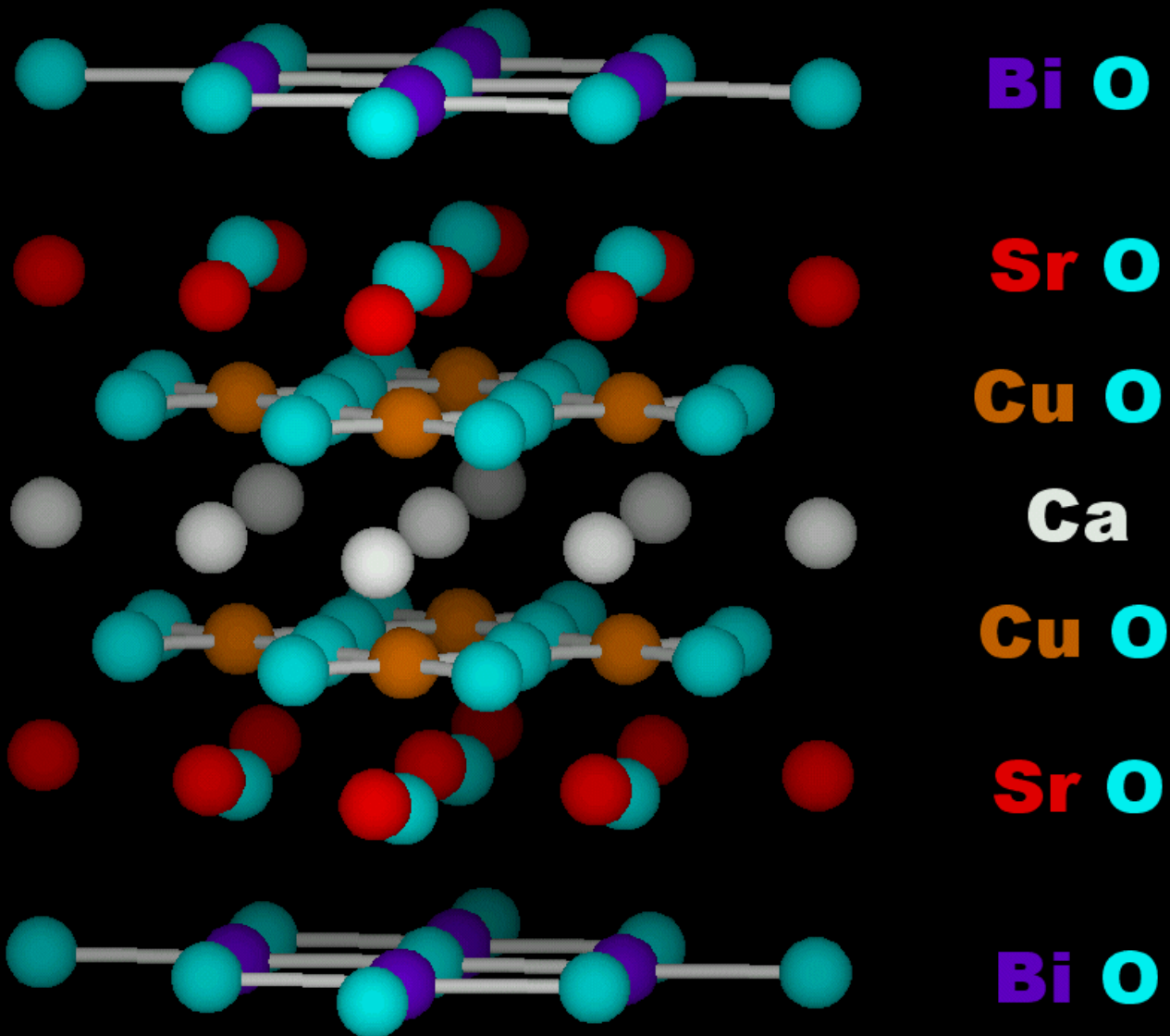
STM Head





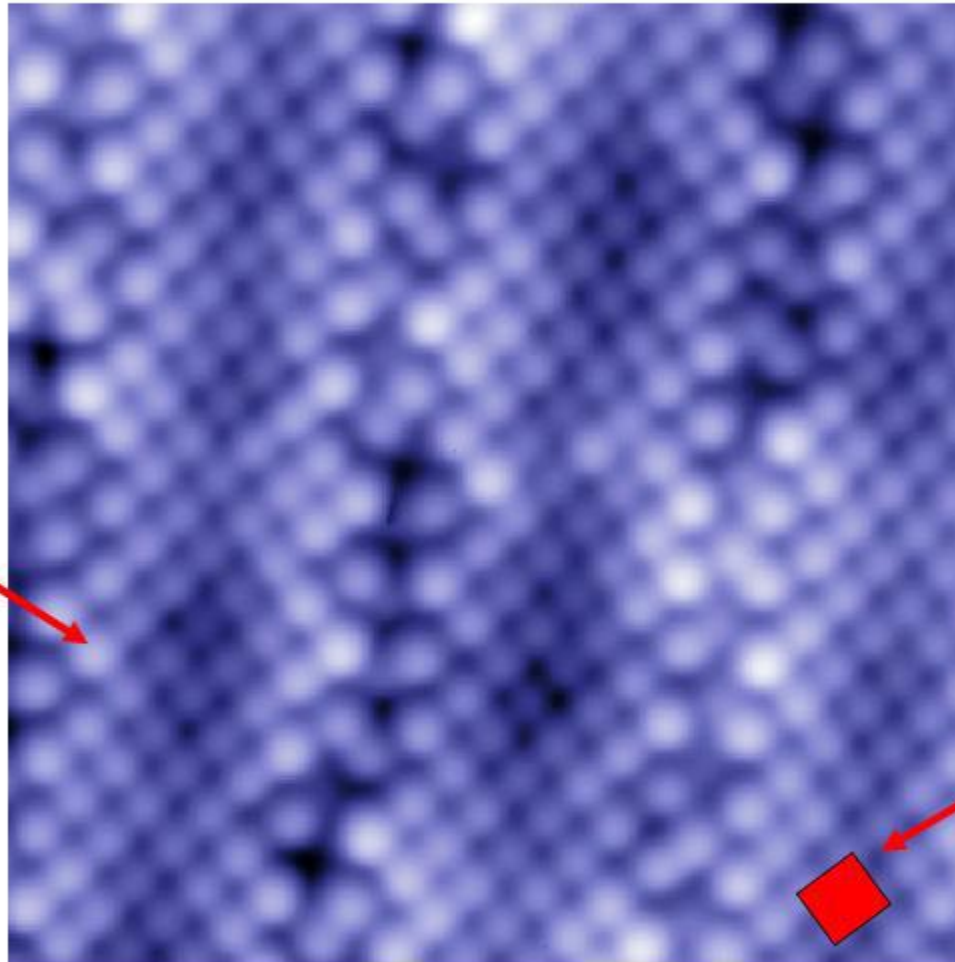
BSCCO

Bi-2212



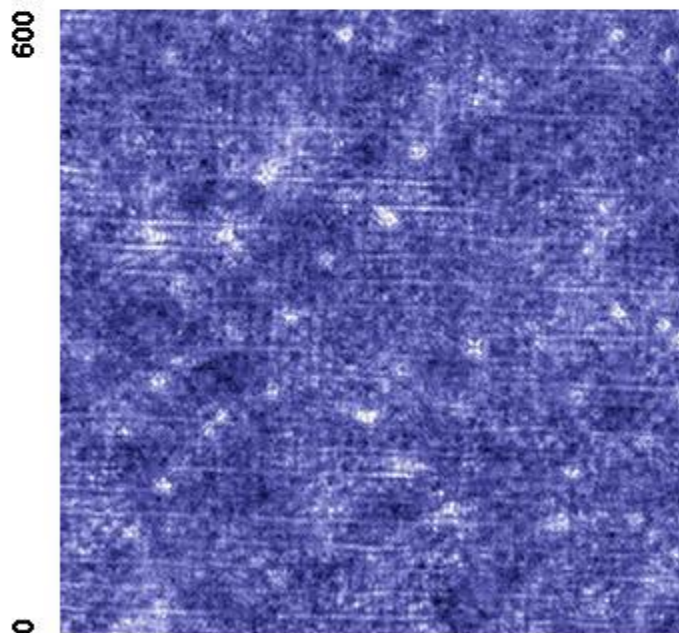
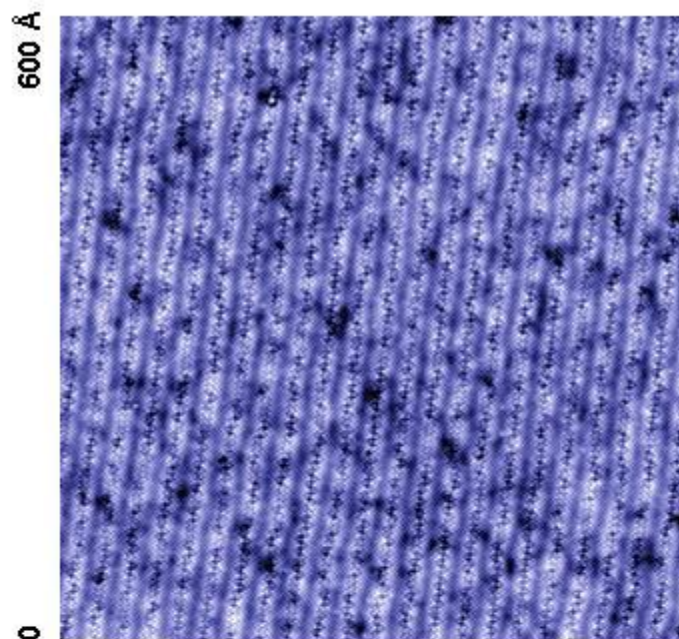


Each bright spot  
is a Bi atom --  
Cu atom is about  
5Å below



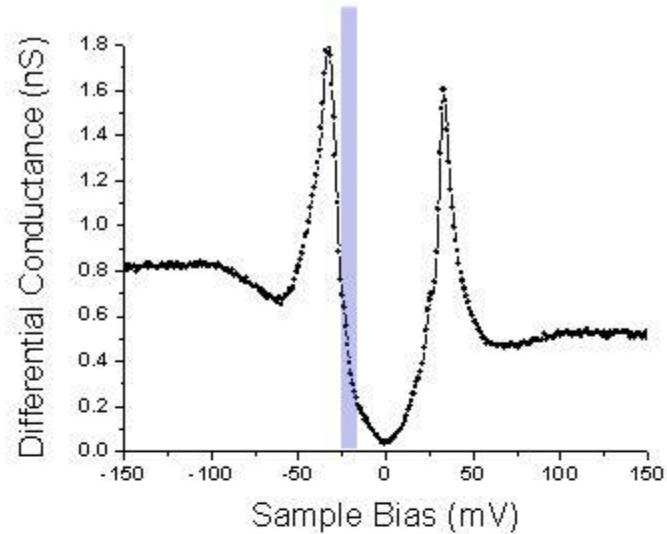
Size of  
 $\text{CuO}_2$   
unit-cell

**T = 4.2K, B = 7T**  
**100pA, -100mV**

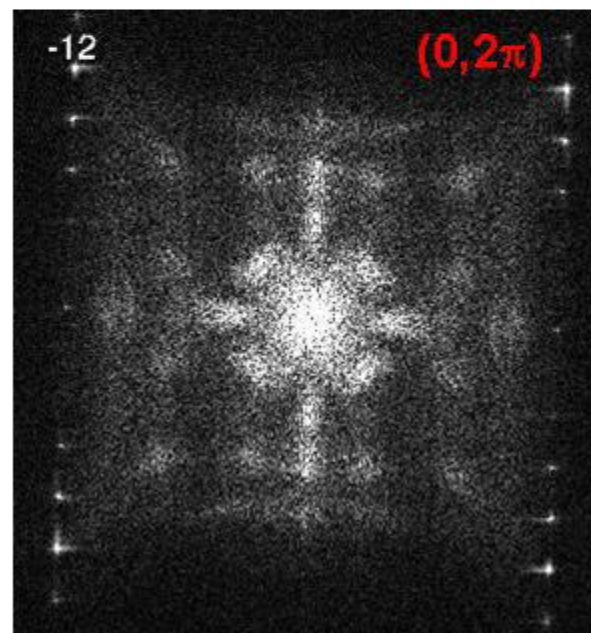


$g(\vec{r}, E = -12 \text{ meV})$

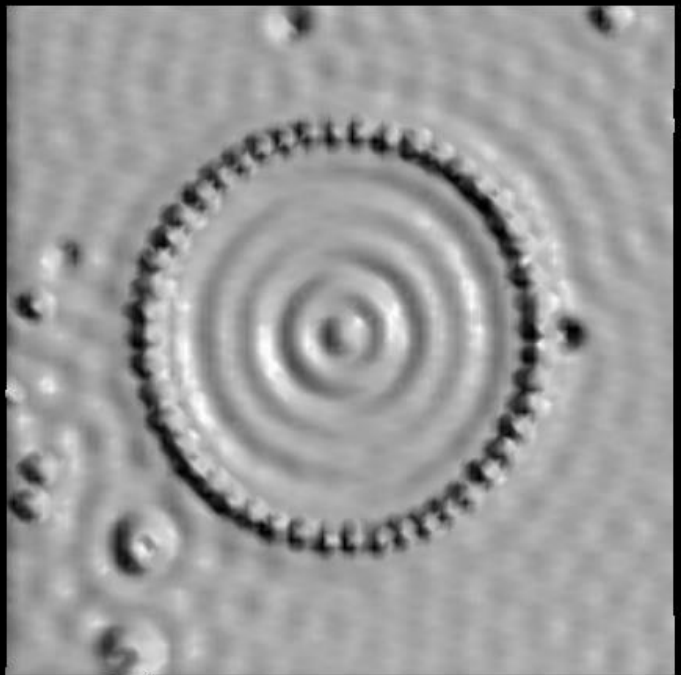
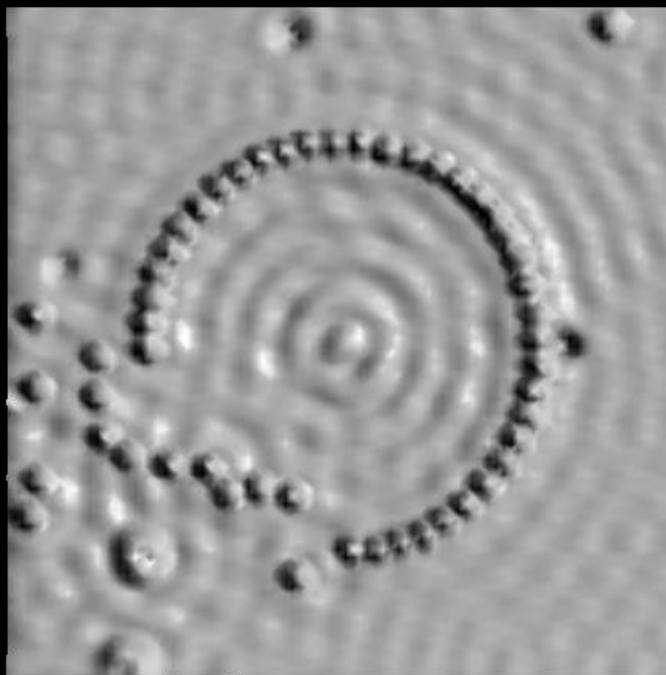
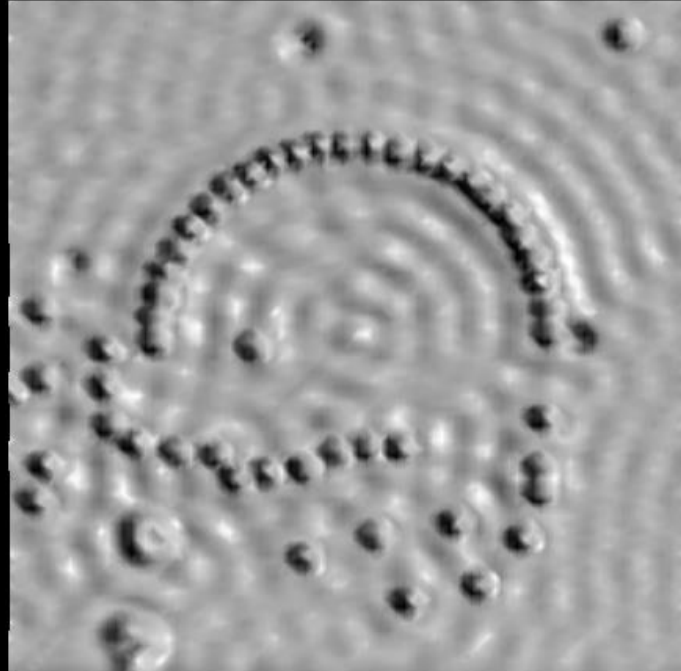
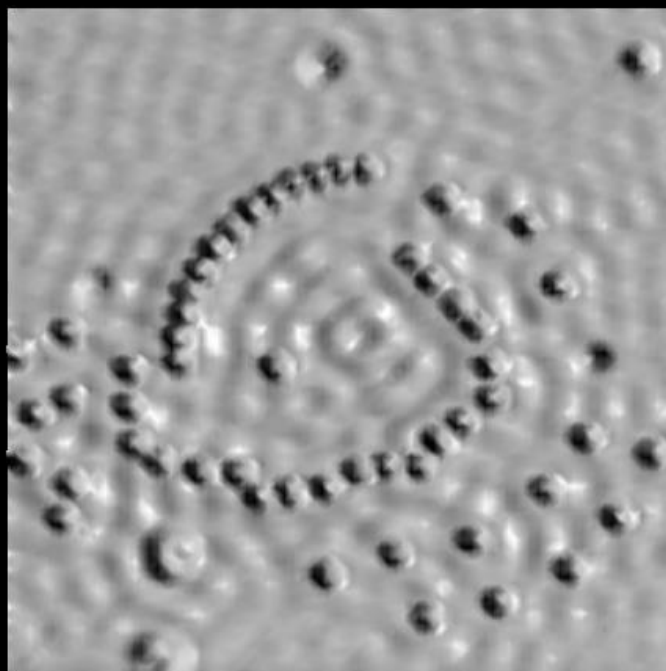
FFT shows  
q-vector of  
LDOS  
modulations



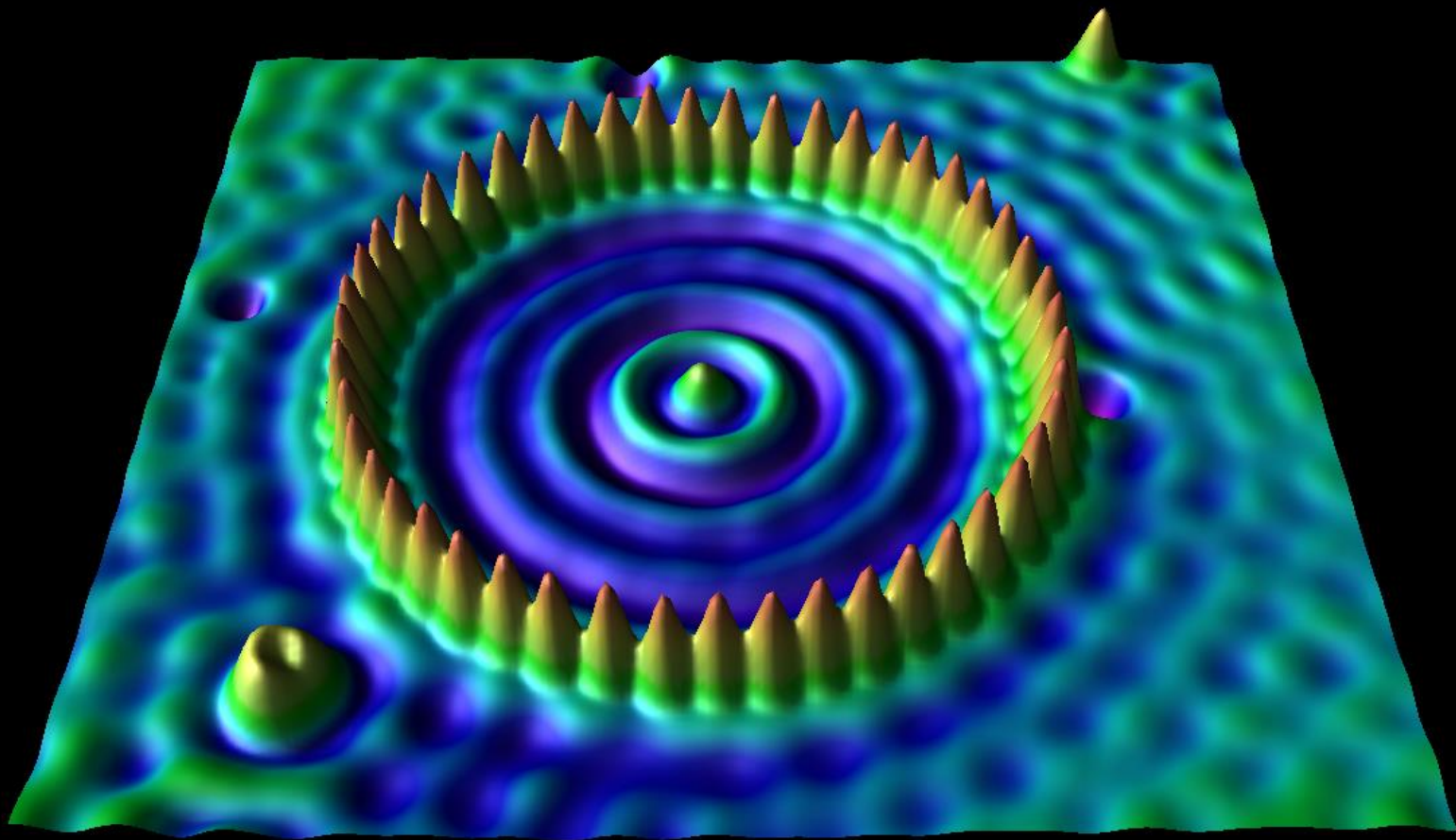
Point  $dI/dV \equiv g(\vec{r}, E)$  Spectrum



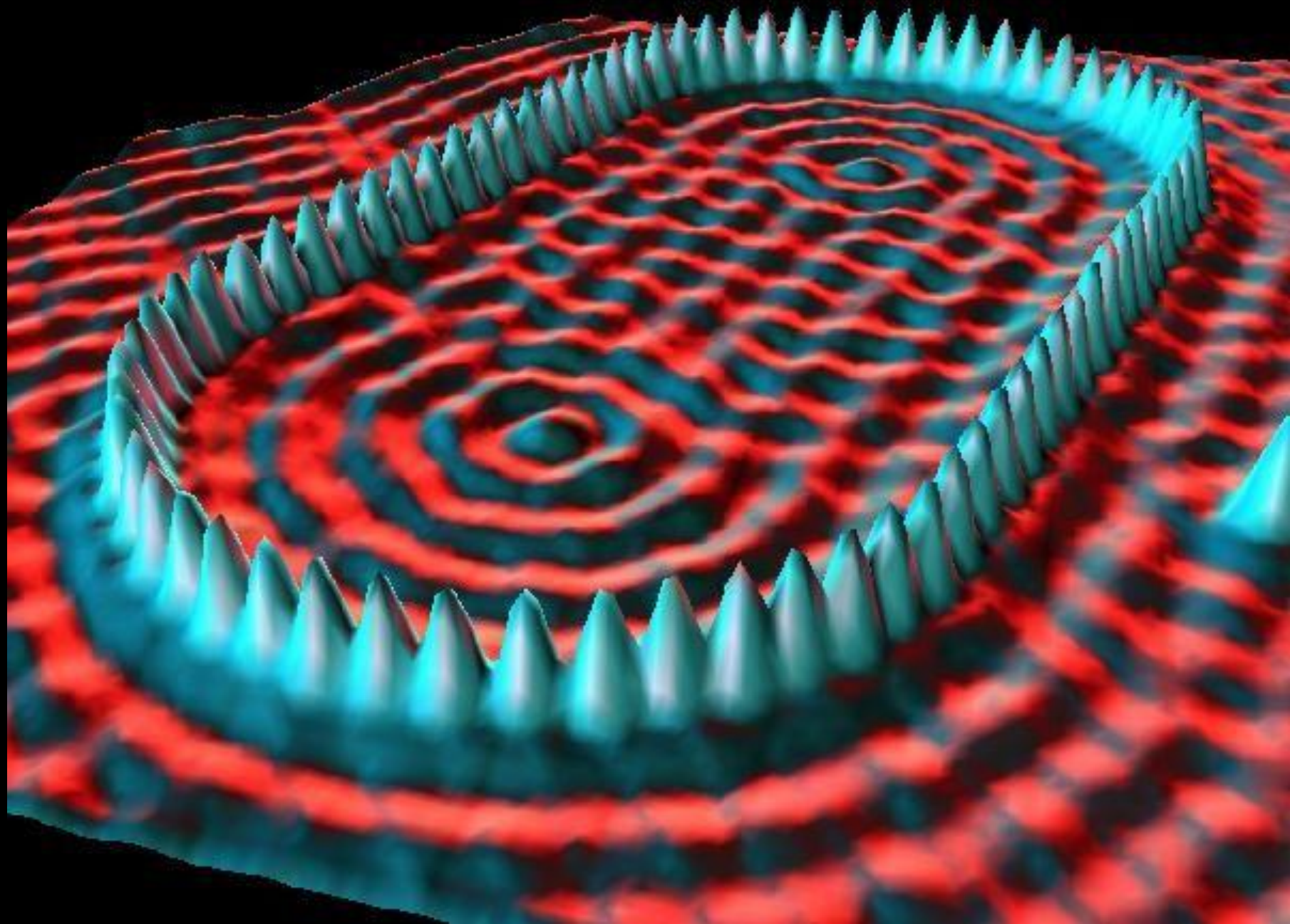
$g(\vec{q}, E = -12 \text{ meV})$



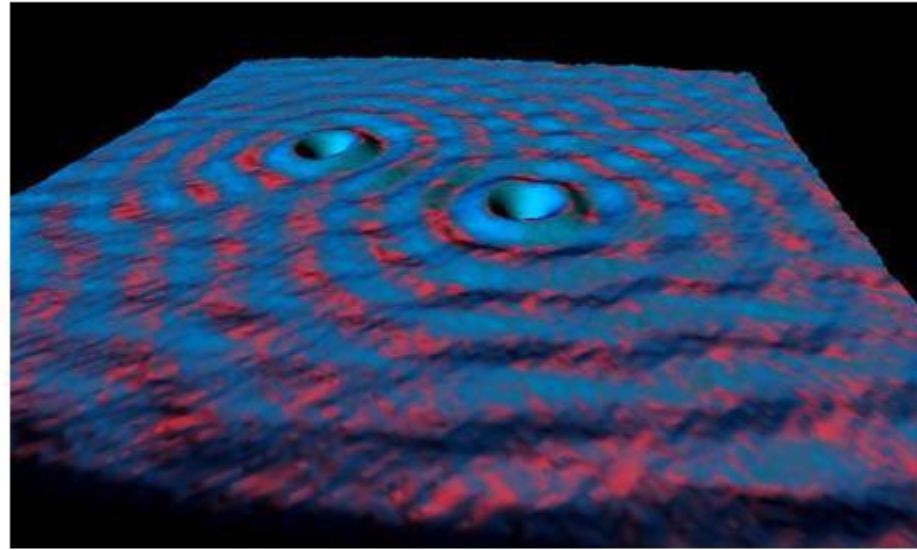








# Quasiparticle Interference at Impurity Atoms

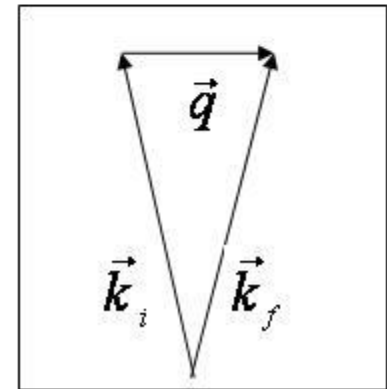


Crommie, Lutz & Eigler, *Nature* **363**, 524 (1993)

- Mixing of  $|\vec{k}_1\rangle$  and  $|\vec{k}_2\rangle$  by scattering creates interference term, with wavevector

$$\vec{q} = \vec{k}_2 - \vec{k}_1$$

- Interference results in modulations in LDOS with wavelength  $\lambda = \frac{2\pi}{|\vec{q}|}$



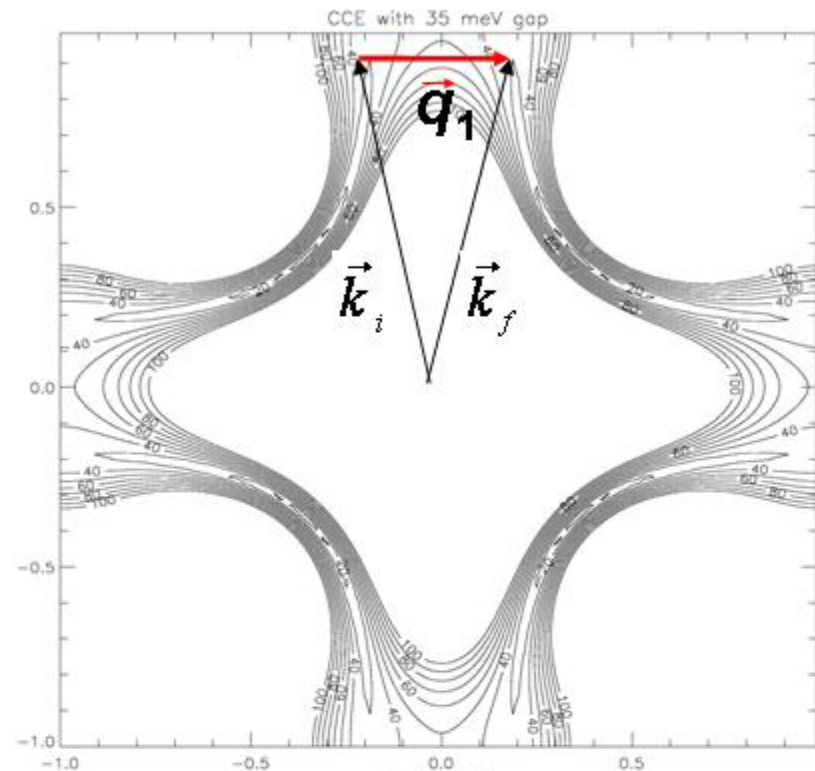
# Simplified Model of Cuprate QP Scattering

If scattering potential is  $V(\vec{r})$   
 $\Rightarrow$  each Fourier component  $V(\vec{q})$   
will cause elastic scattering  
between initial state  $|\vec{k}_i\rangle$  and final  
state  $|\vec{k}_f\rangle$  whose momenta differ  
by  $\vec{q}$ .

A simplified model for scattering  
rate  $w_{if}$  is Fermi Golden Rule

$$w_{if} = \frac{2\pi}{\hbar} |\langle \vec{k}_f | V(\mathbf{q}) | \vec{k}_i \rangle|^2 n_f(E)$$

where  $n_f(E)$  is the densities of  
final states.

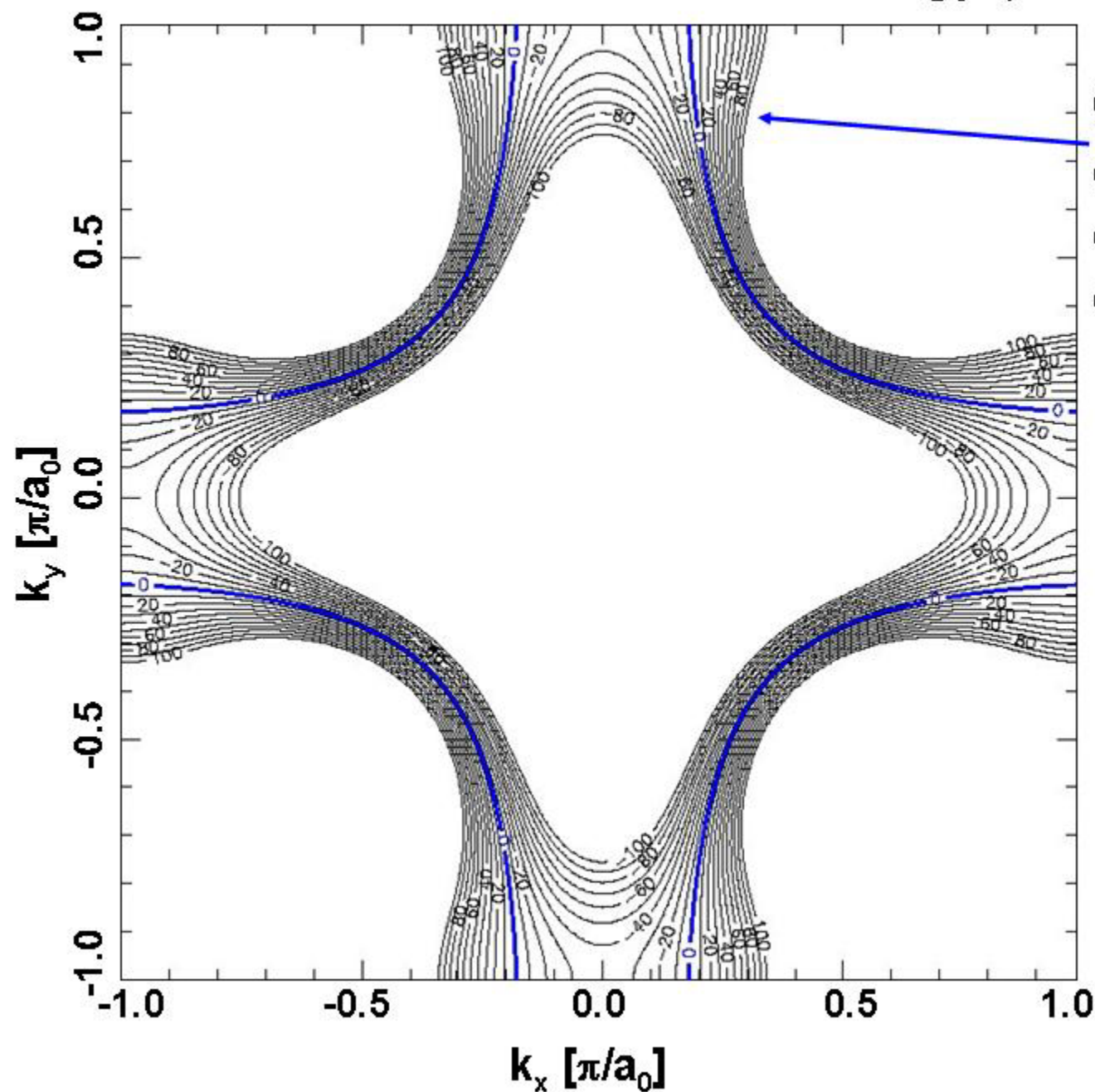


States with highest  $n_f(E)$   
will dominate scattering.

Where are these states in  
momentum space?



# Normal State Contours of Constant Energy (CCE) : Band Structure

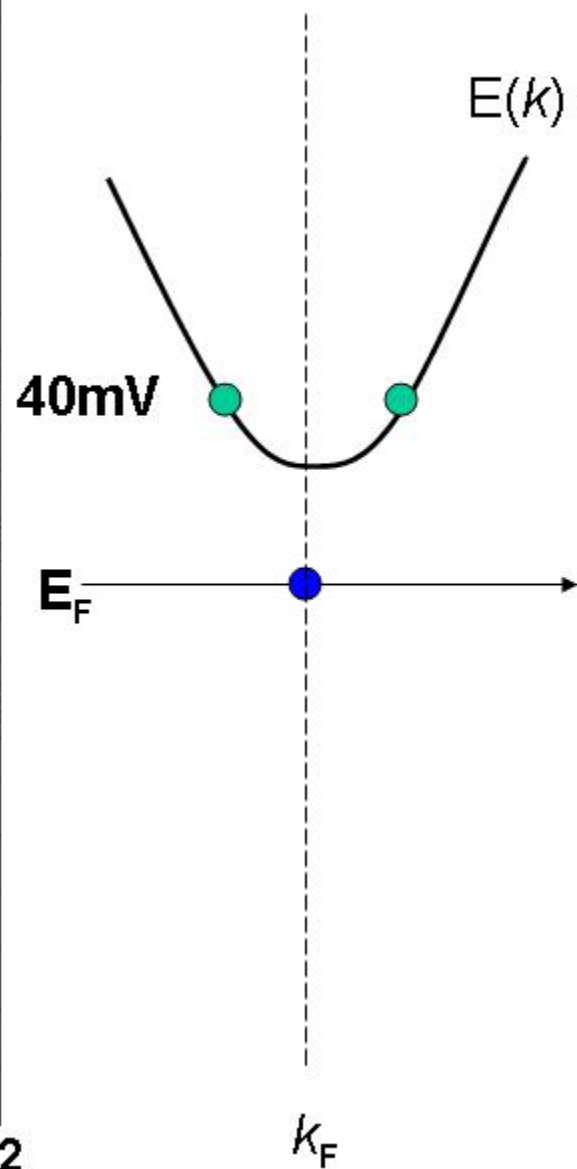
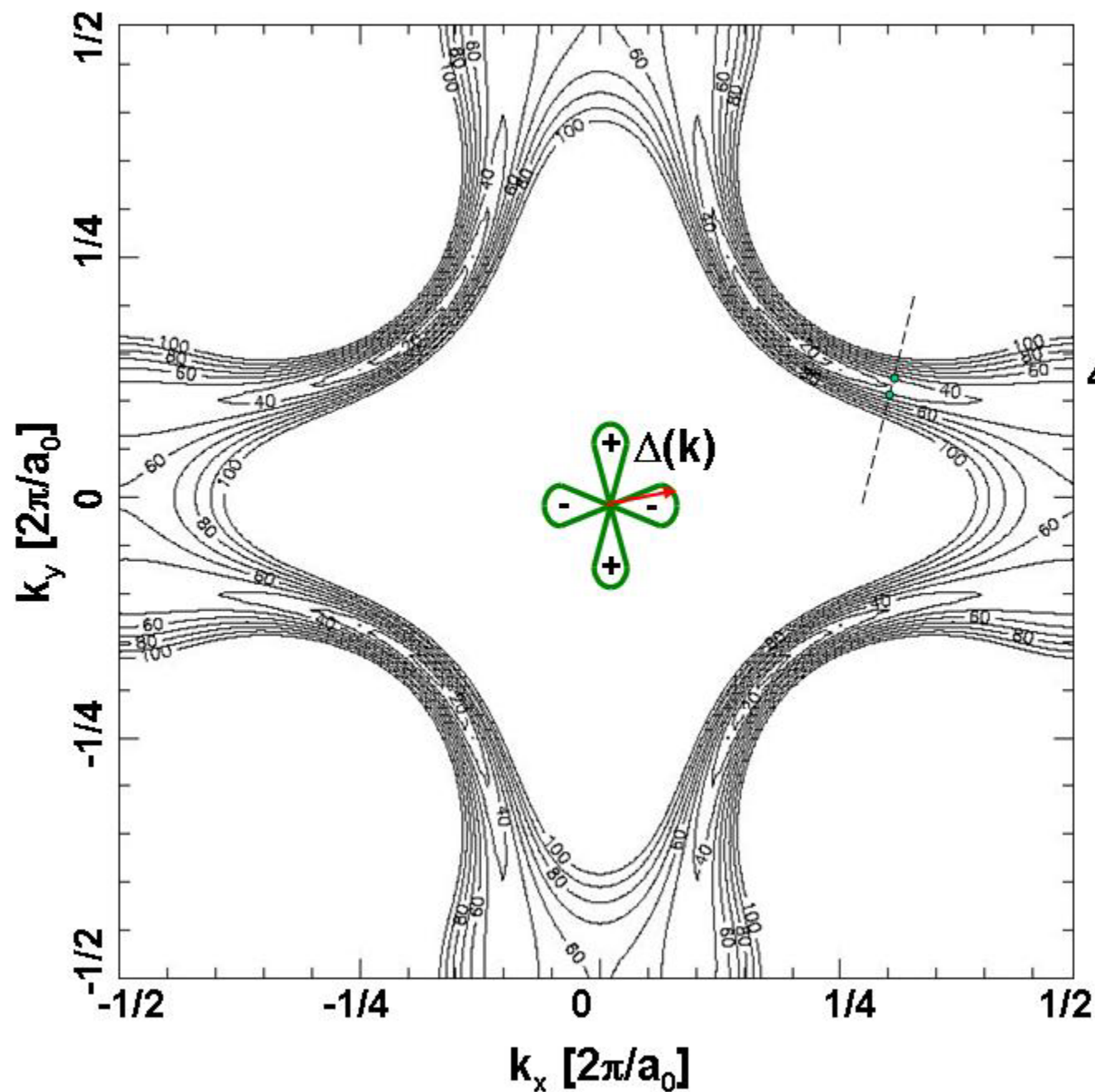


CCE (E): The location in k-space of states with energy E

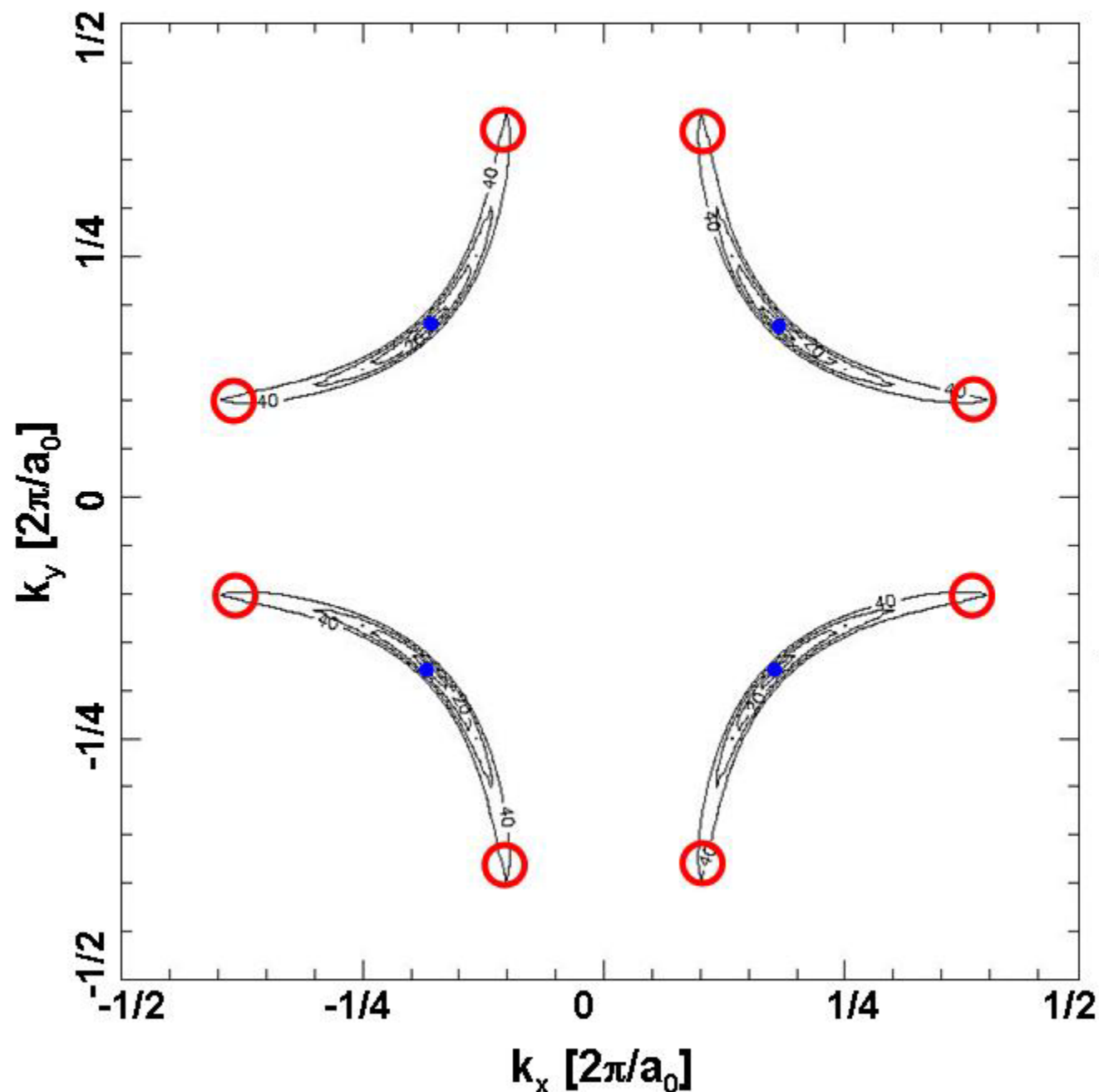
Parameterization:  
M. Norman  
PRB 52, 615  
(1995).

Based on data:  
Ding *et al.*,  
PRL 74, 2784  
(1995).

In the SC state, a gap  $\Delta(\vec{k})$  opens along FS



Octet of regions at ends of 'bananas' have smallest  $dE/|dk|$



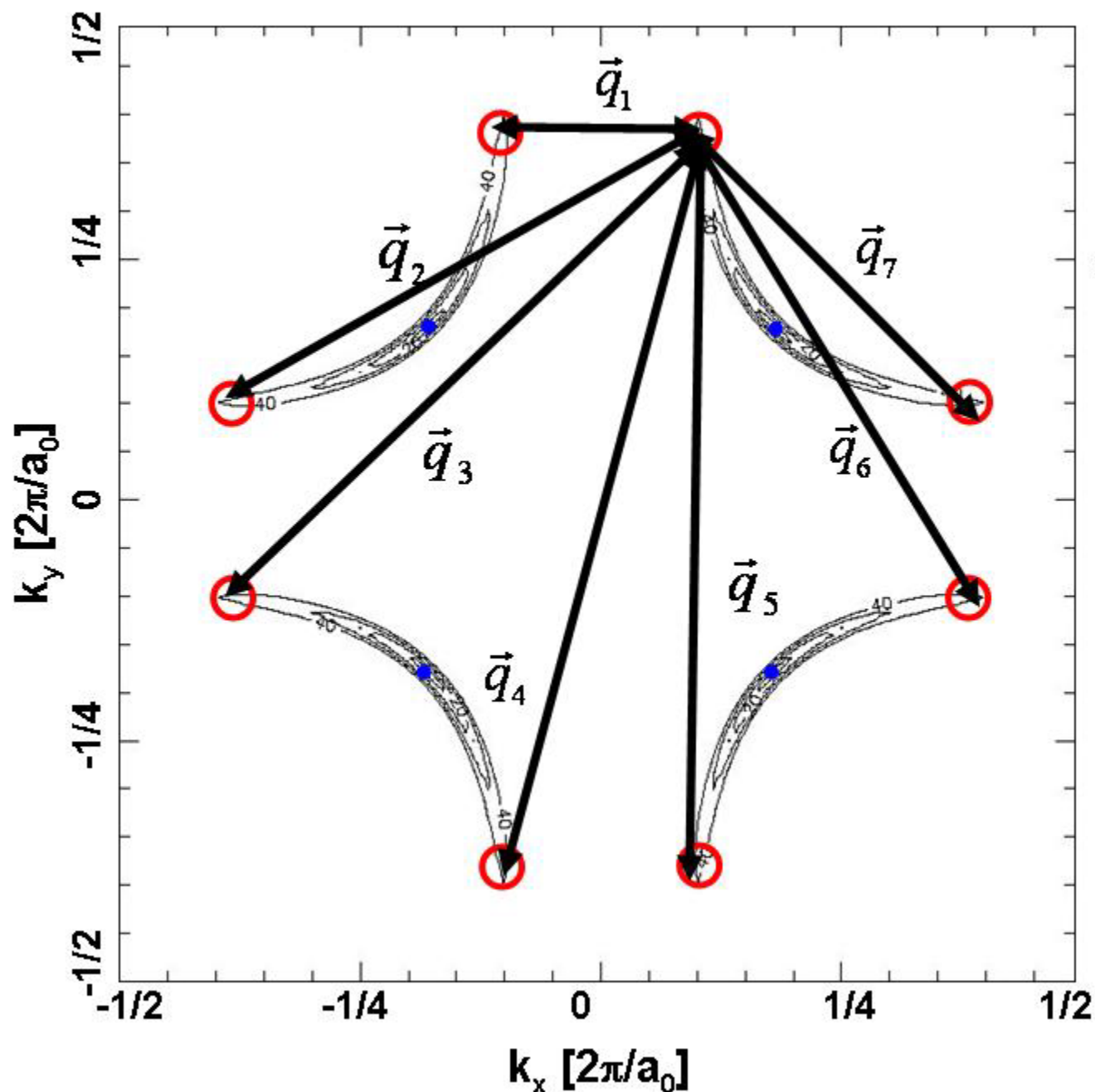
Density of States

$$n(E) = \oint_{E(\vec{k})=E} \frac{1}{|\nabla_{\vec{k}} E(\vec{k})|} d\vec{k}$$



This octet of locations at the tips of the 'bananas' provide maximum contribution to  $n_f(E)$  and thus dominate elastic scattering processes.

Octet of regions at ends of 'bananas' have smallest  $dE/|dk|$



Density of States

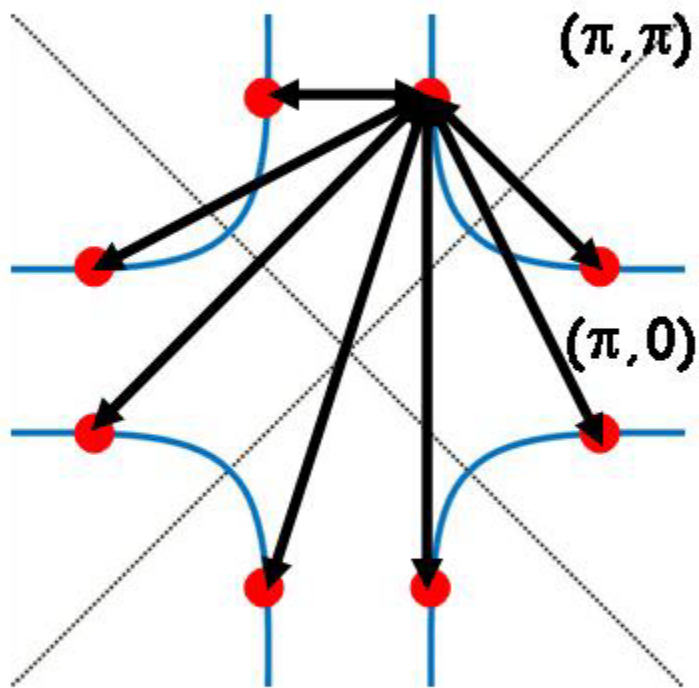
$$n(E) = \oint_{E(\vec{k})=E} \frac{1}{|\nabla_{\vec{k}} E(\vec{k})|} d\vec{k}$$



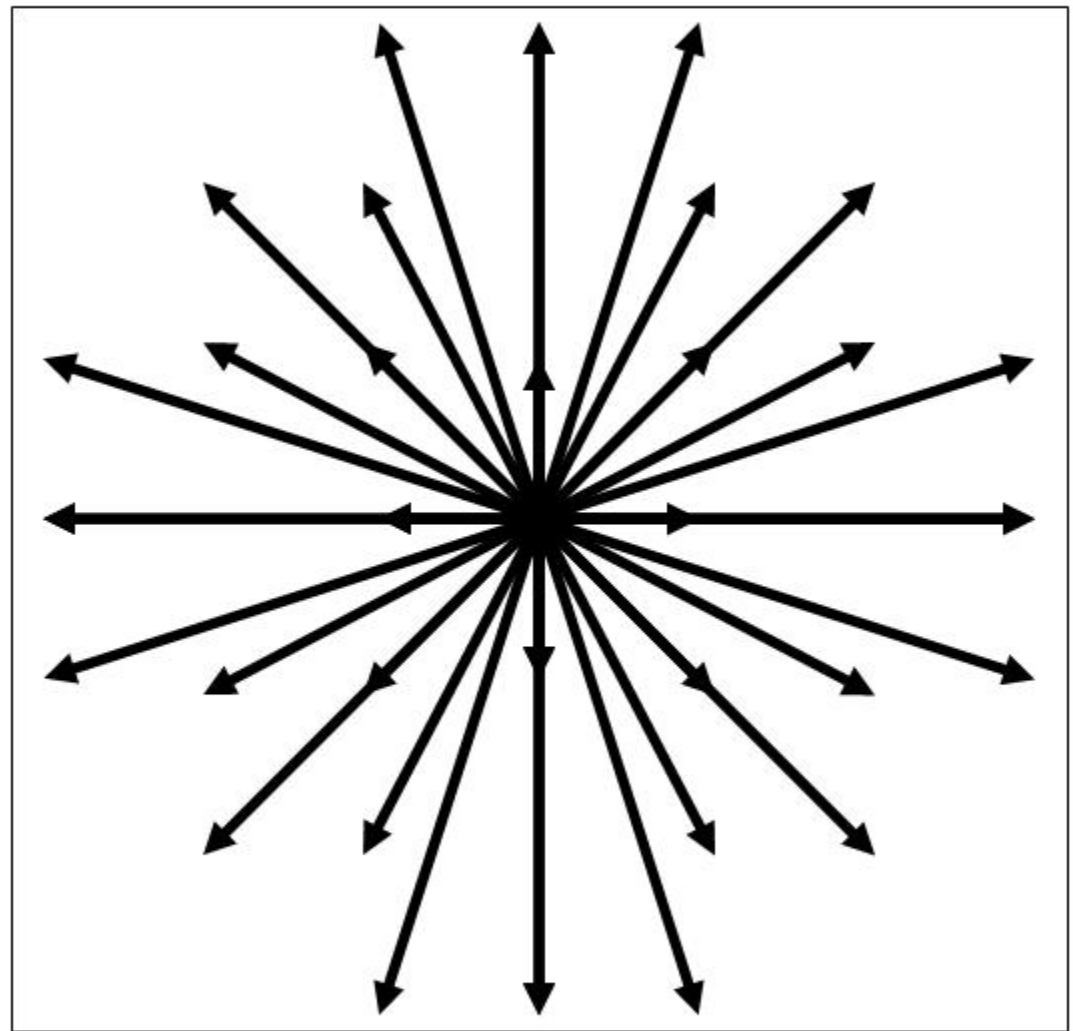
Characteristic set of quasiparticle interference wavevectors which is different at each energy.



Expected energy dependence of these sets of q-vectors



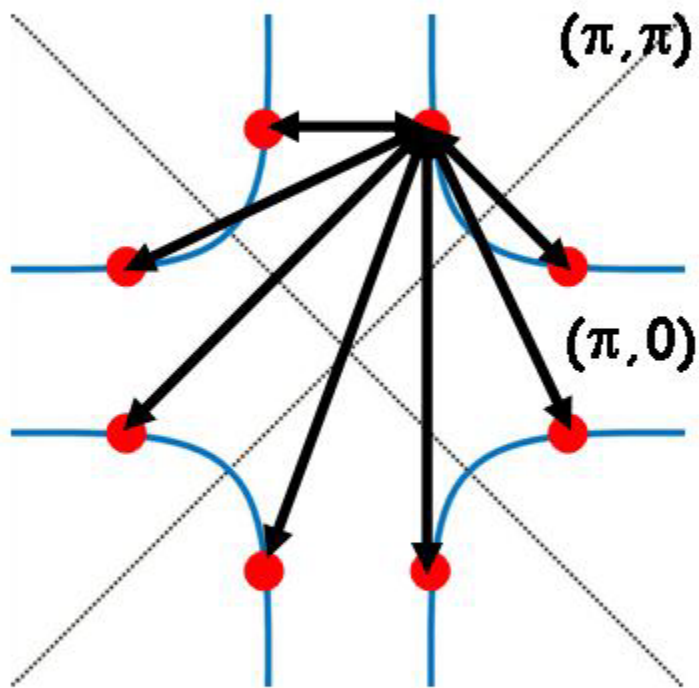
**k-space**



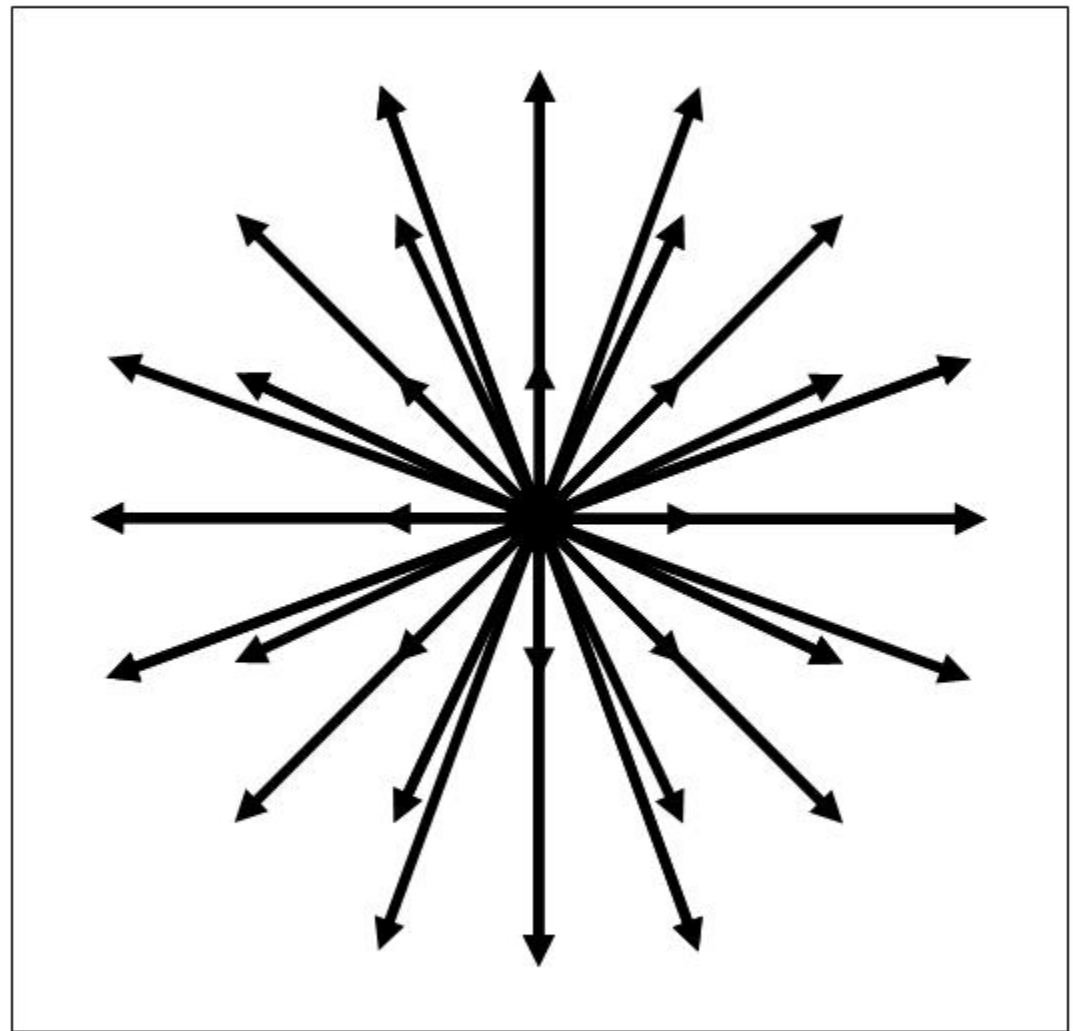
**q-space**



Expected energy dependence of these sets of q-vectors

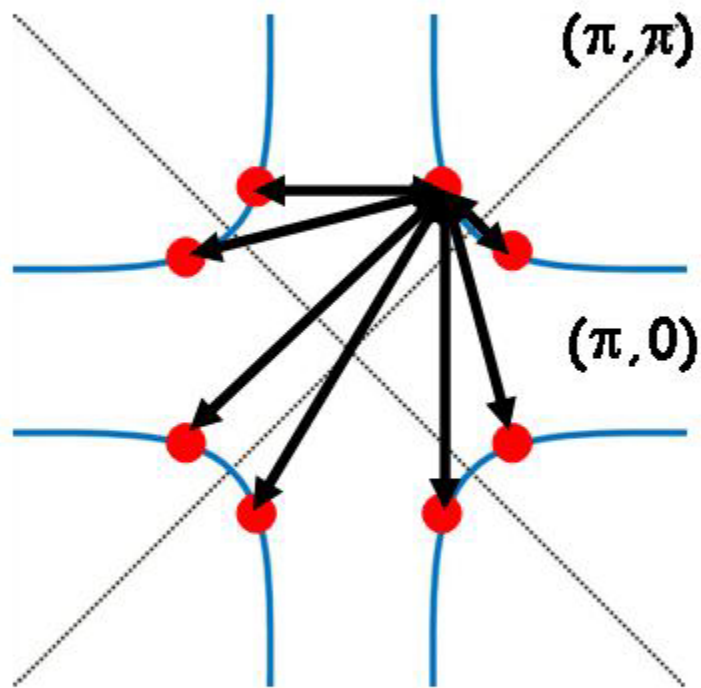


**k-space**

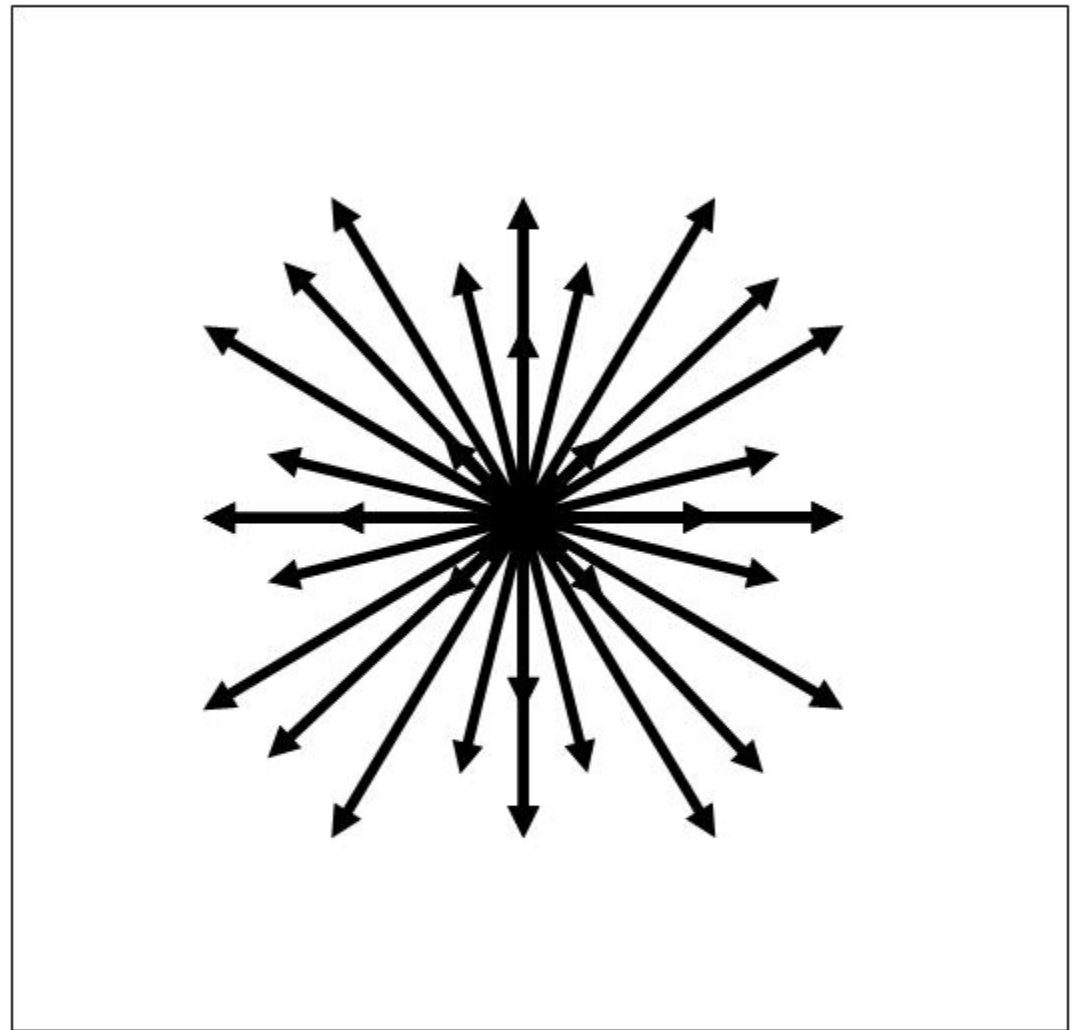


**q-space**

Expected energy dependence of these sets of q-vectors

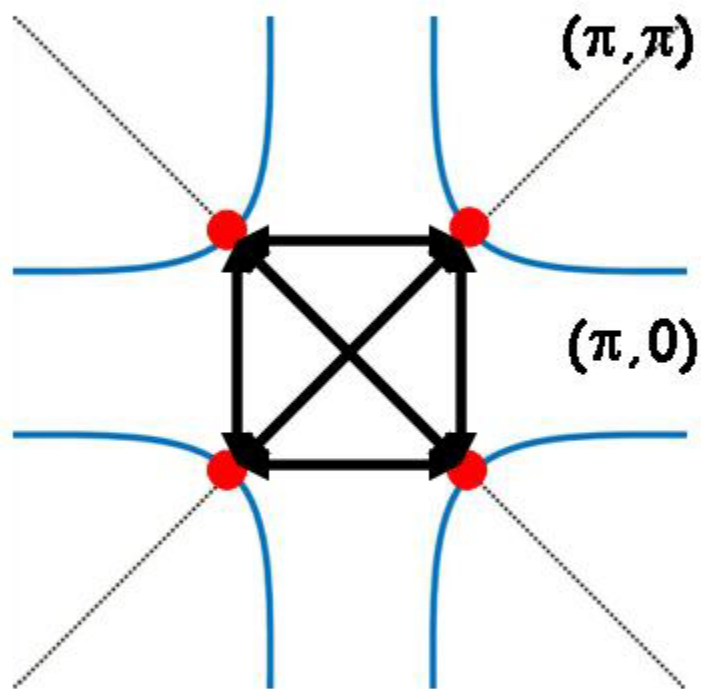


**k-space**

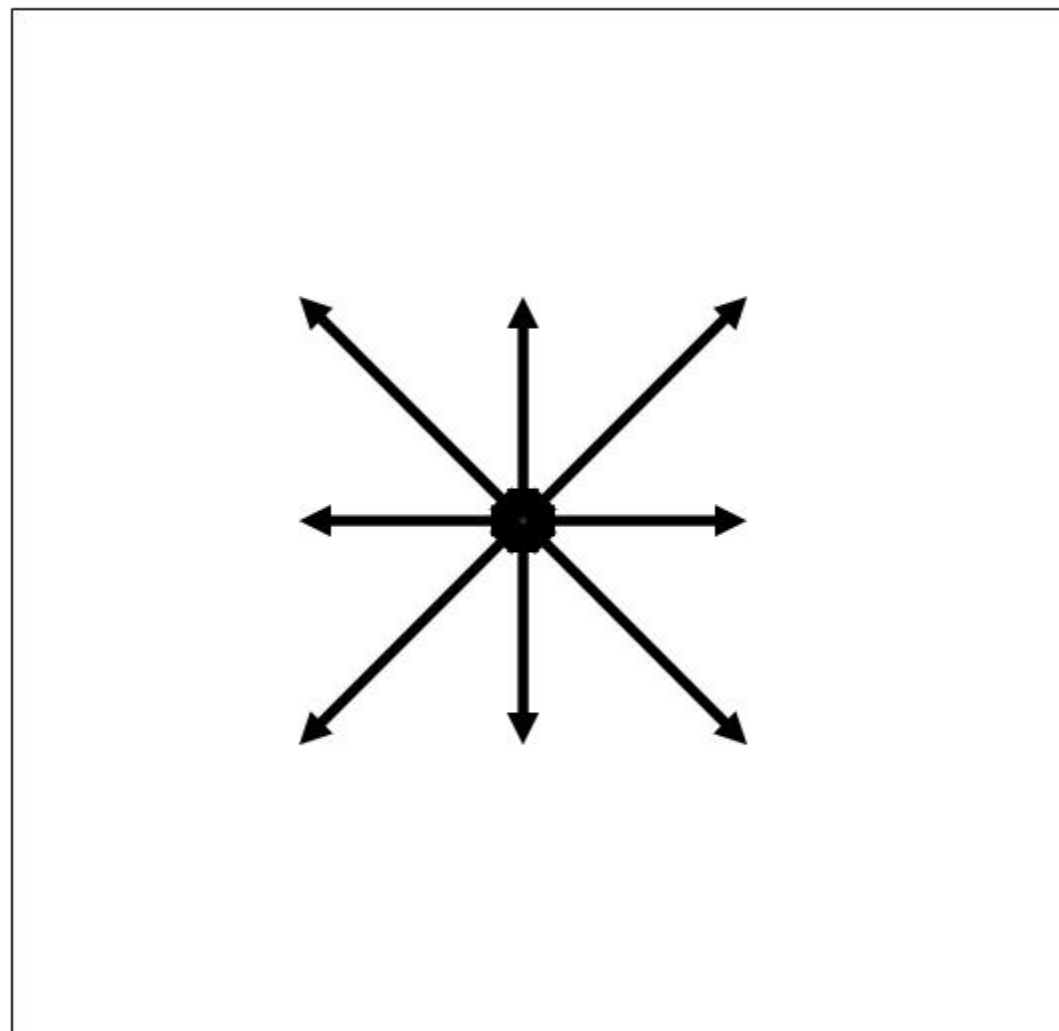


**q-space**

Expected energy dependence of these sets of q-vectors



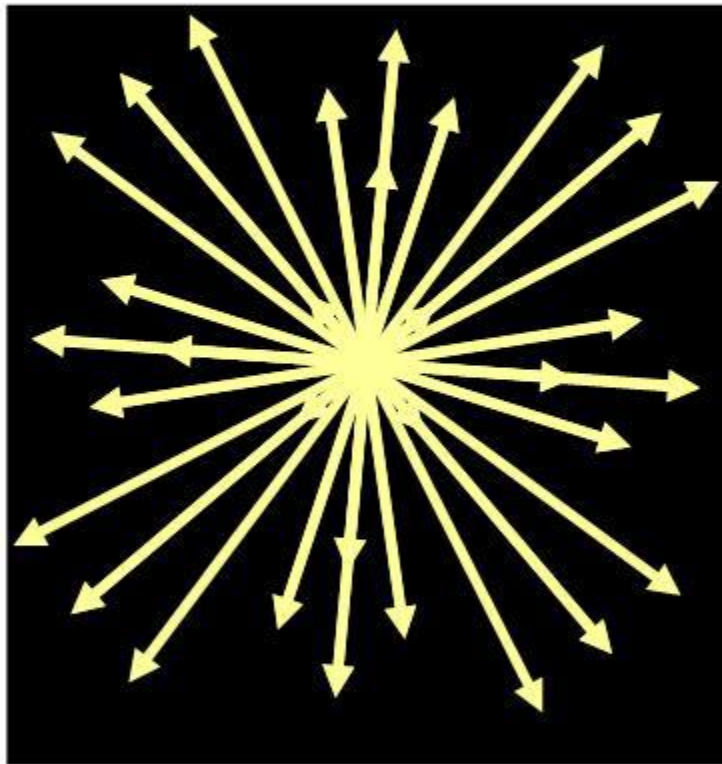
**k-space**



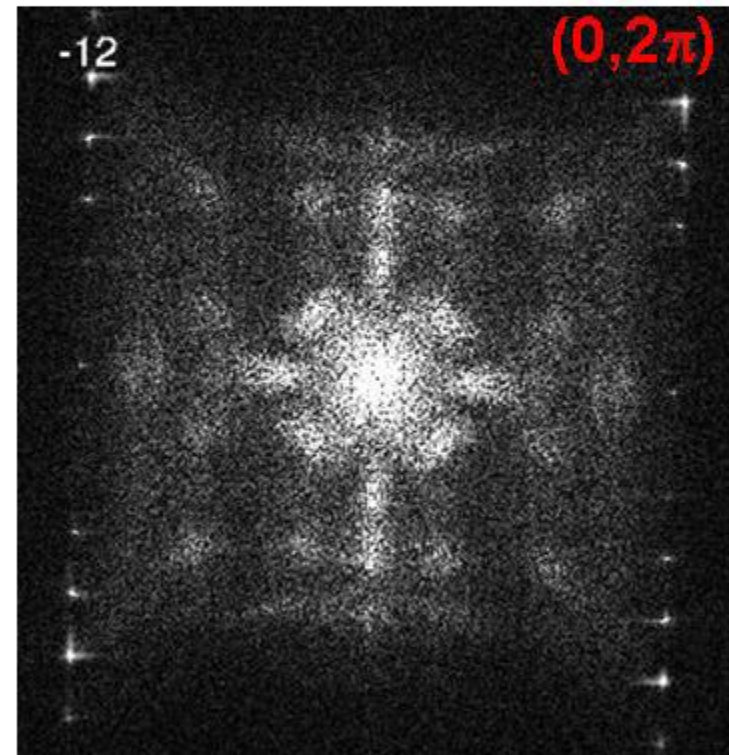
**q-space**

# The set of modulations is consistent with 'octet' model

Octet-model expected set of q's.



Measured set of q's.

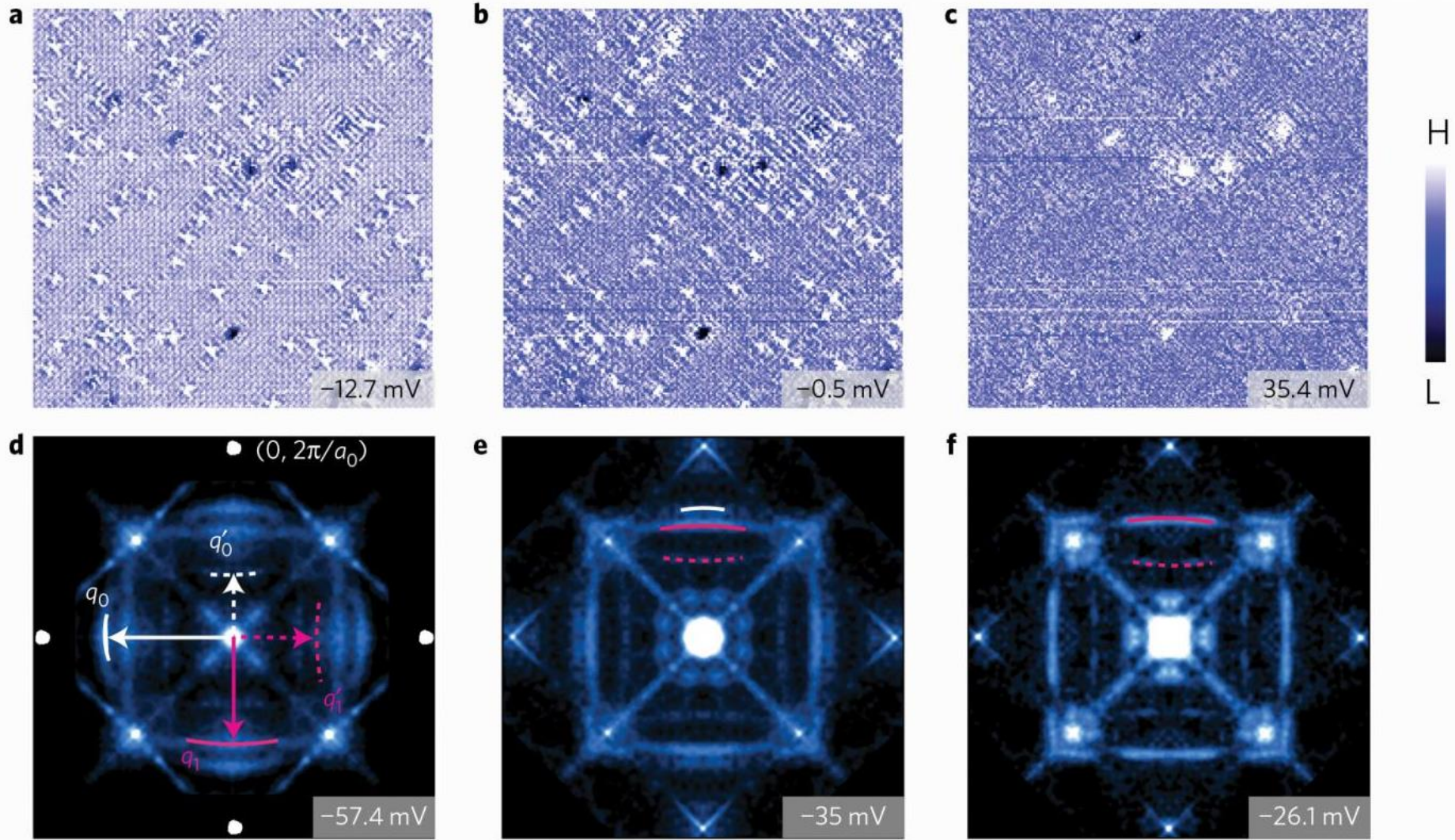


q-space

-12mV

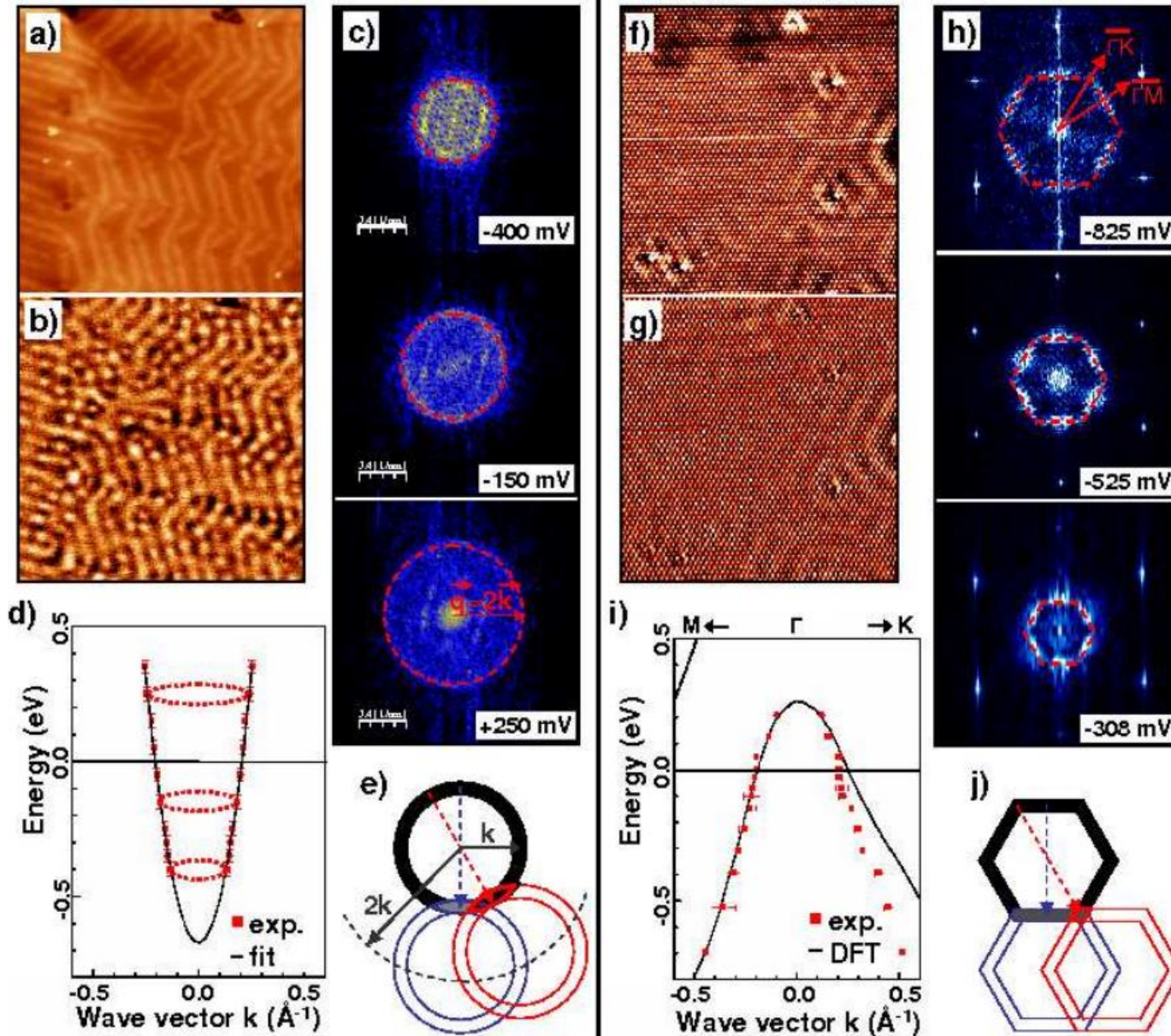


# Sr2RuO4 Quasiparticle Interference



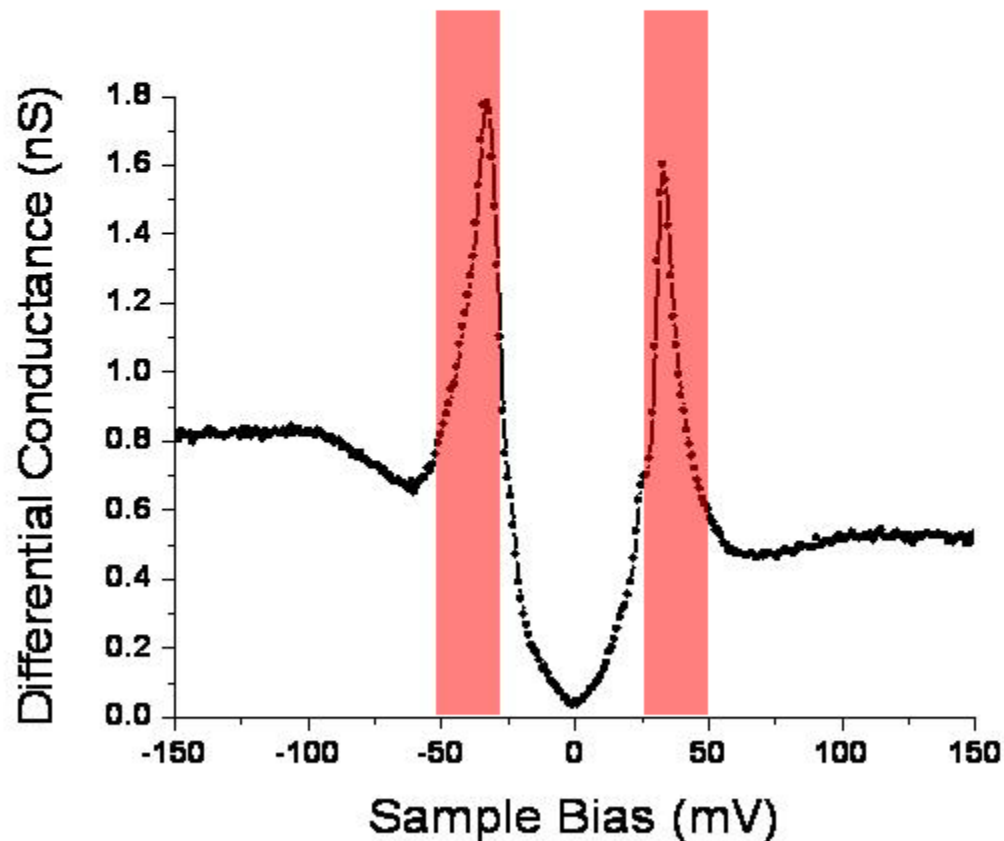


# Au(111)



# Gap Edge

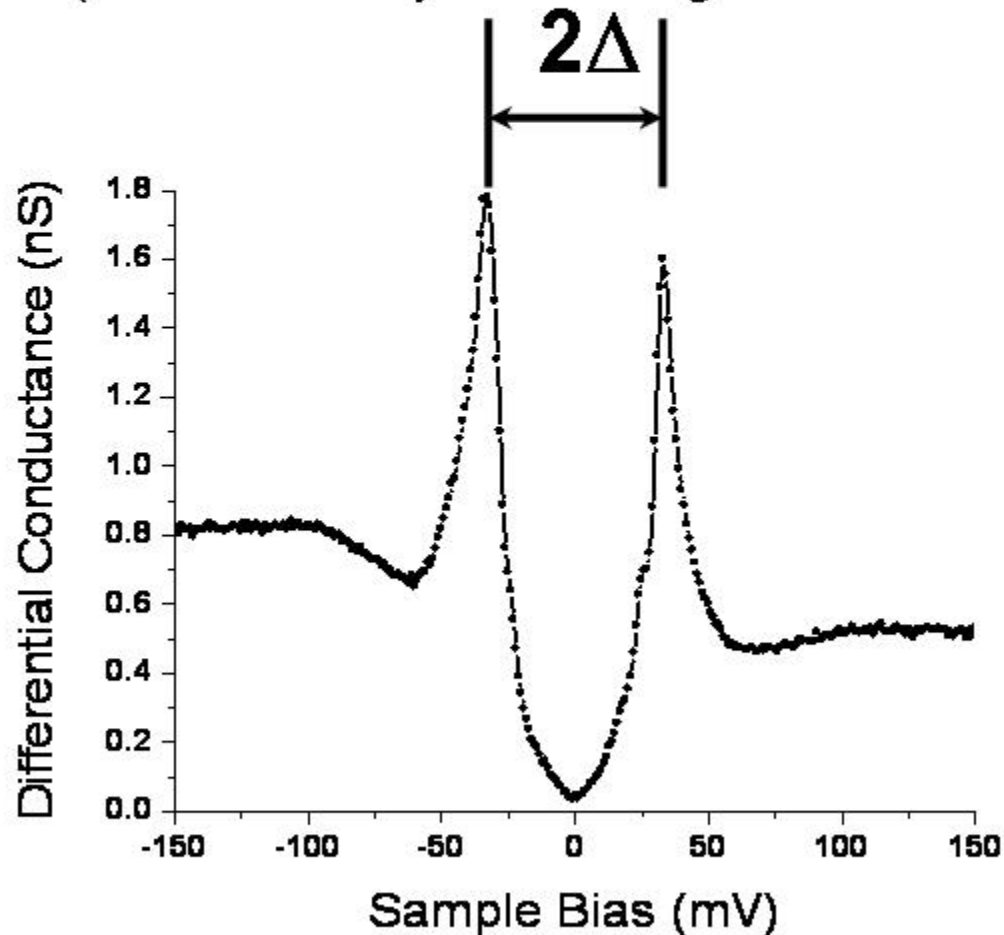
What about the higher energy states?



# GapMap:

## Map of $\Delta$ as a function of location

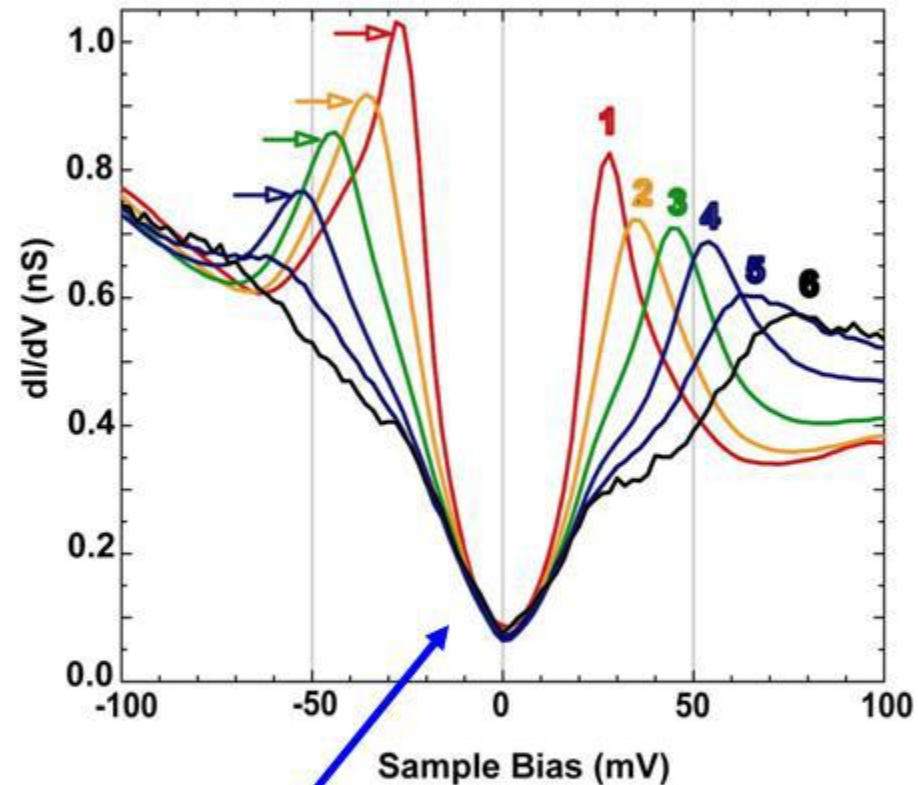
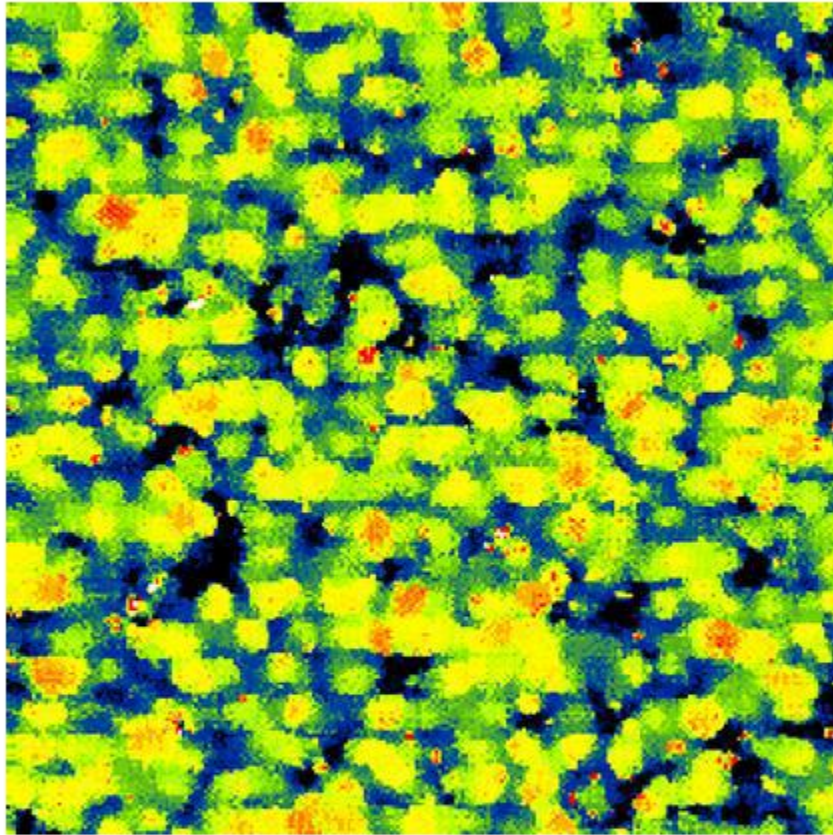
(on an atomically resolved/registered surface )





# Gapmap disorder not effecting the nodes

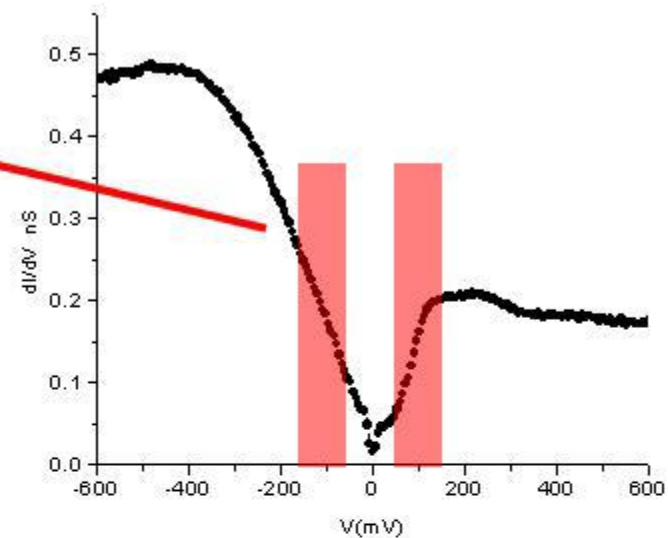
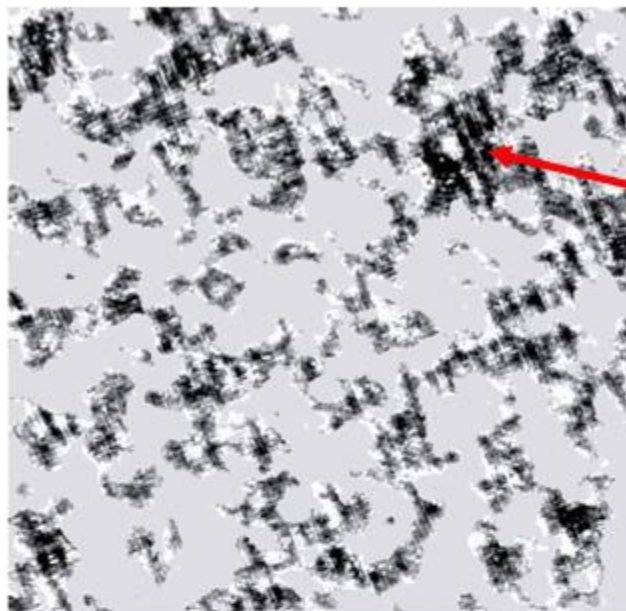
~600 Å



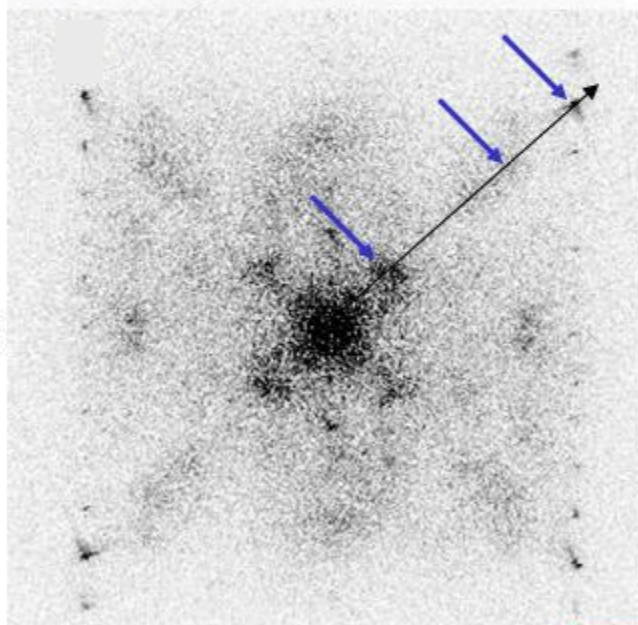
You can see that the low energy, nodal, quasiparticles are rather homogeneous

# 'Checkerboard' charge modulations @ ZTPG Spectra

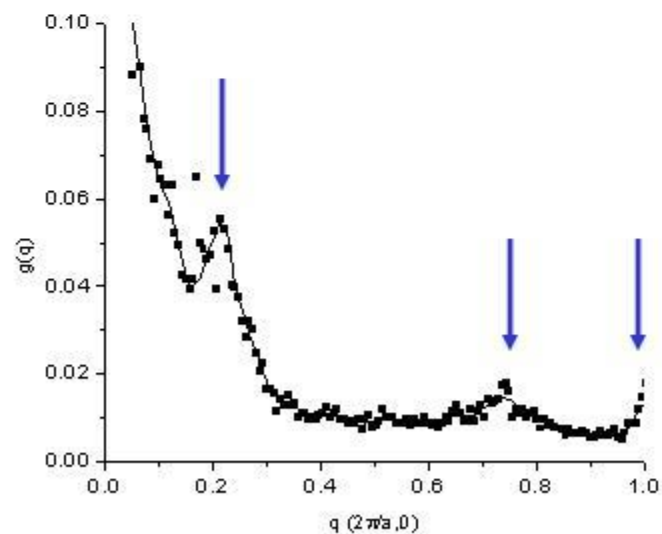
600 Å



FFT



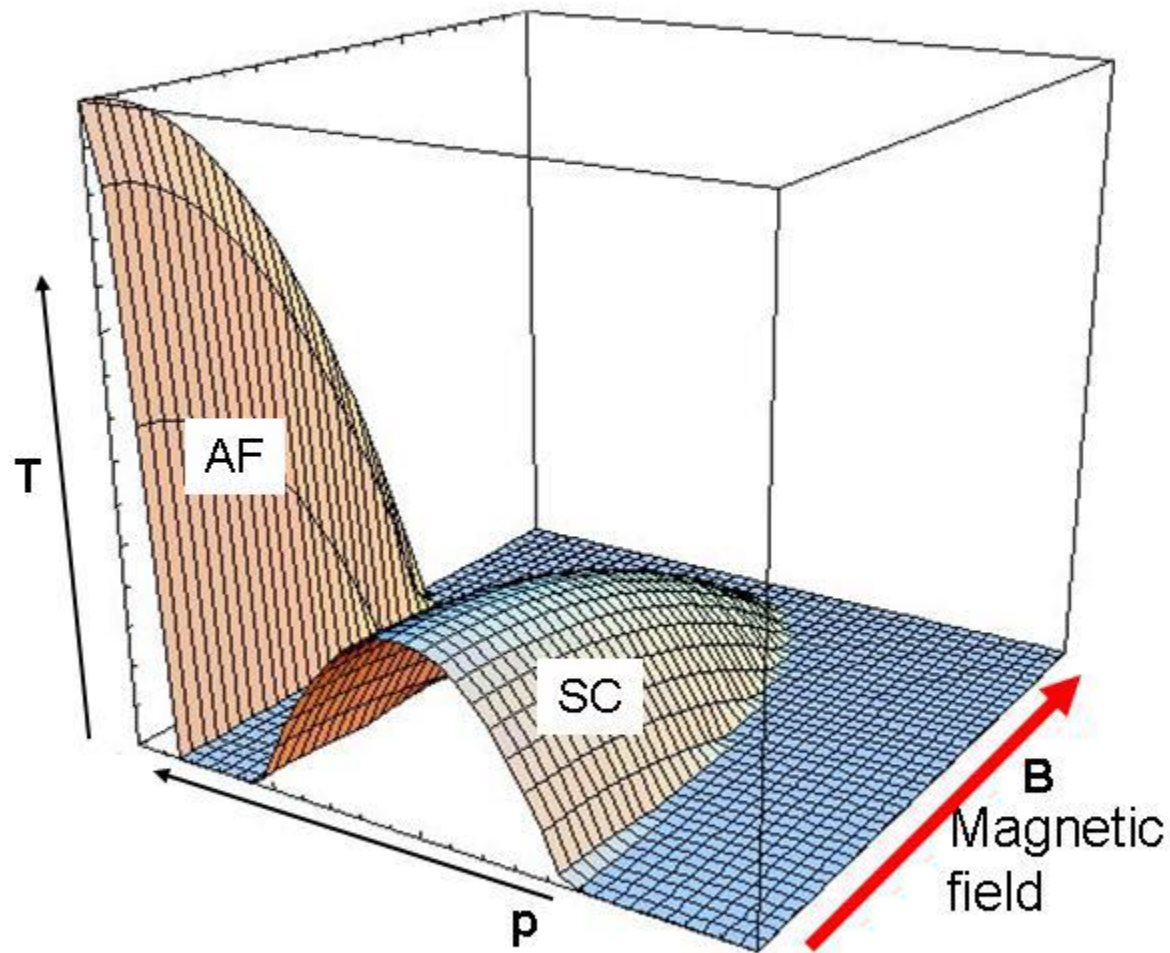
$g(\vec{q}, \Delta E)$



$(\pi, 0)$



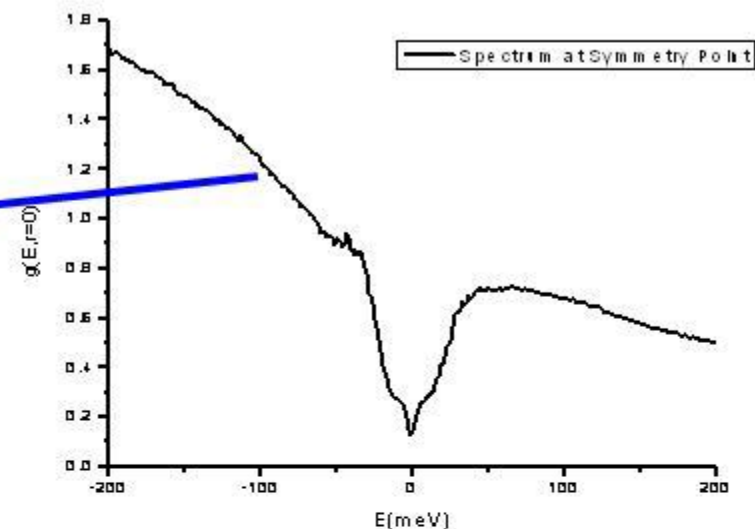
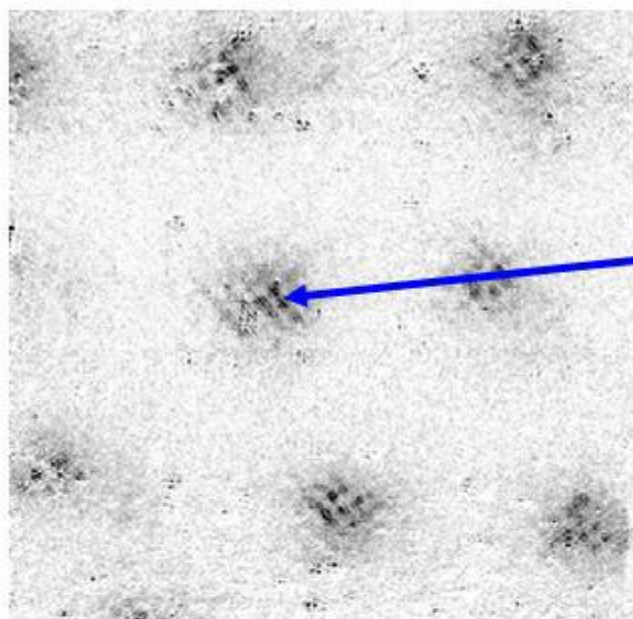
# 1. Bi-2212 Studies: High Field



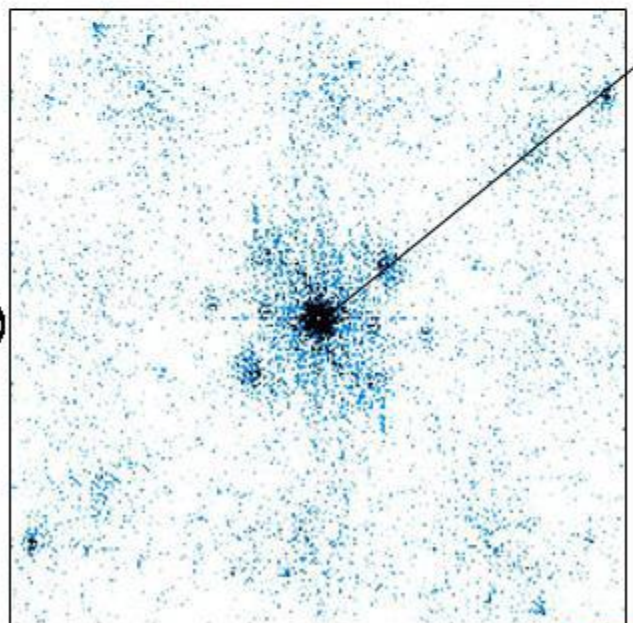
Superconductivity is destroyed in the  
cores of quantized vortices

# $\sim 4a_0$ 'Checkerboard' LDOS modulations @ V-core Spectra

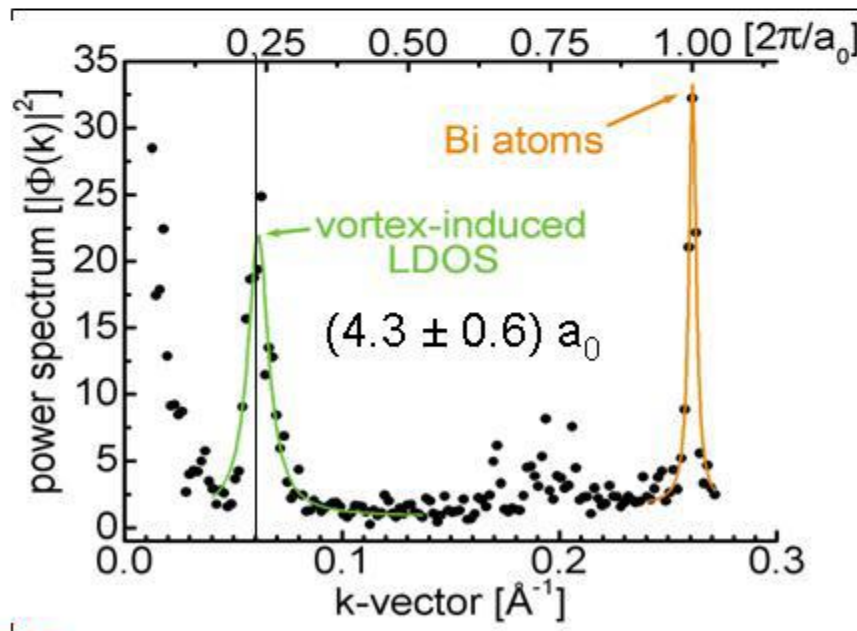
560 Å



FFT



$g(\vec{q}, \Delta E)$



$(\pi, 0)$

# Way to ARPES

Impurity scattering hypothesis:  $|FS(\mathbf{r})| = ACA(\mathbf{k})$

Definition of autocorrelation:  $ACA(\mathbf{k}) = \int A(\mathbf{k})A(\mathbf{k} + \mathbf{q})d\mathbf{k}$

Another definition through  
the Fourier transform  
(Wiener-Khinchin theorem):

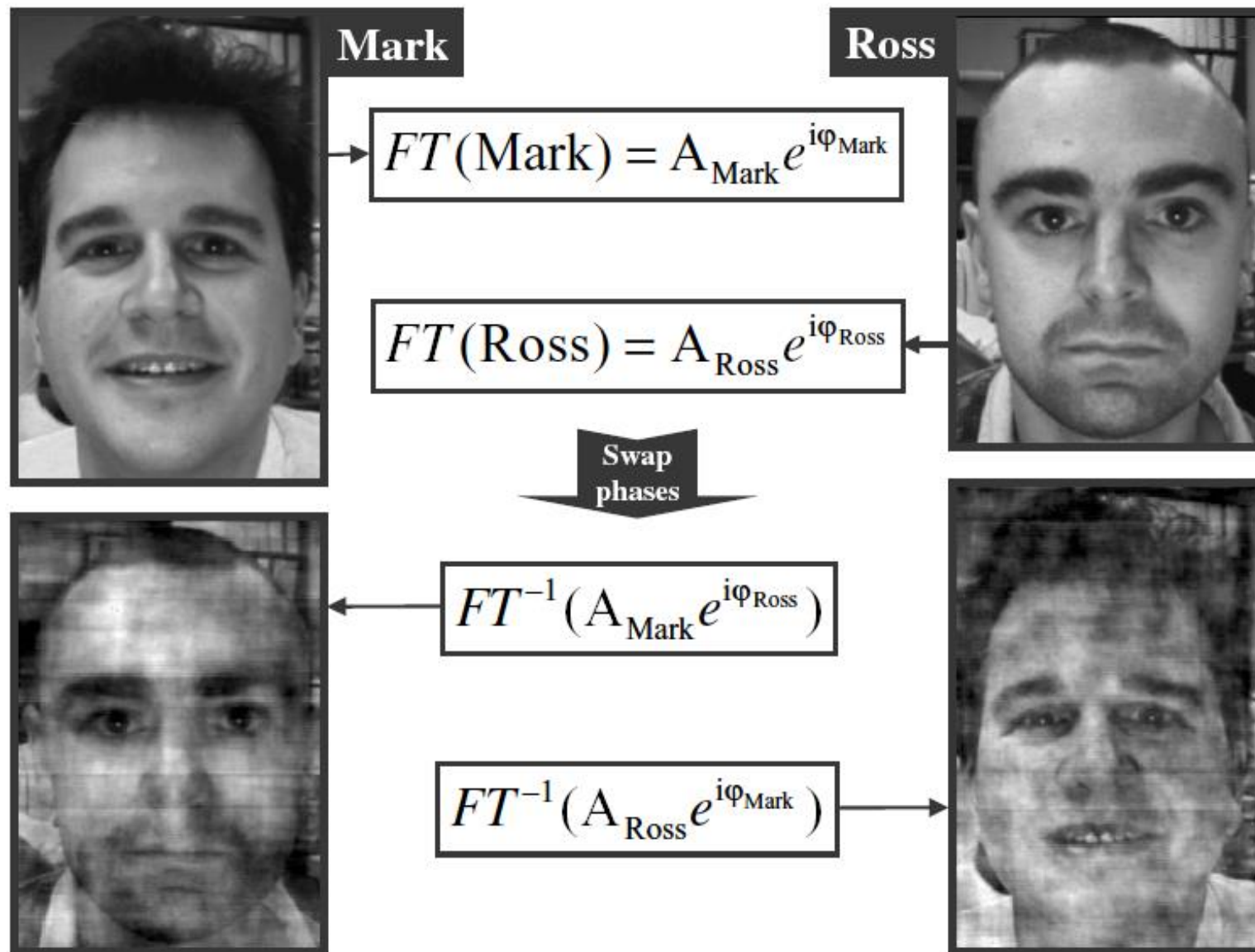
$$ACA(\mathbf{k}) = F|FA(\mathbf{k})|^2$$

$$|FA(\mathbf{k})| = \sqrt{F|FS(\mathbf{r})|} = R(\rho)$$

Phase retrieval algorithm:  $A(\mathbf{k}) = \text{PRA} \sqrt{F|FS(\mathbf{r})|}$

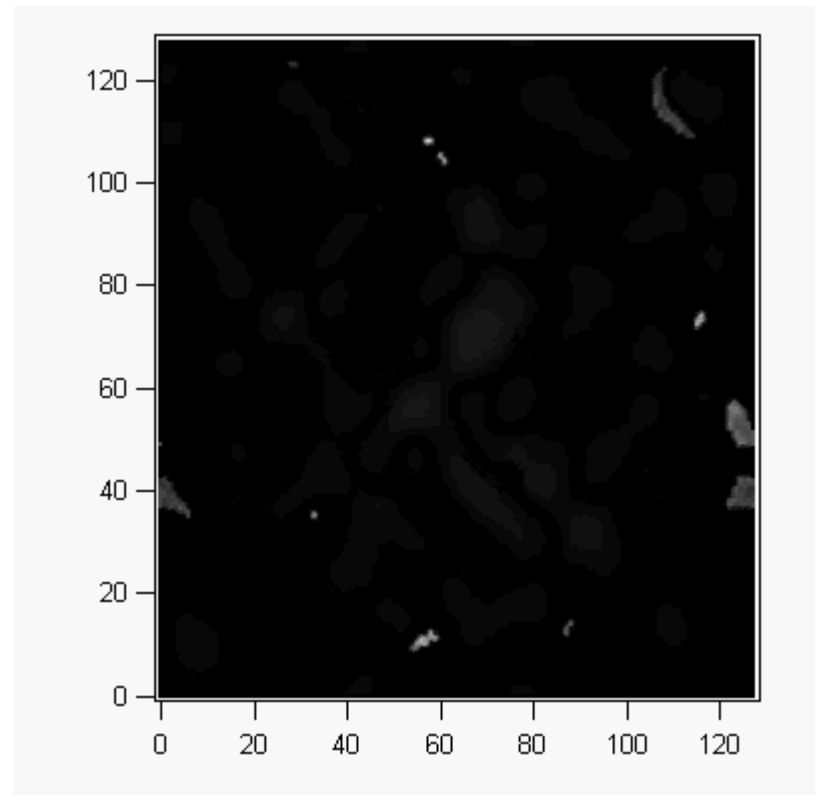
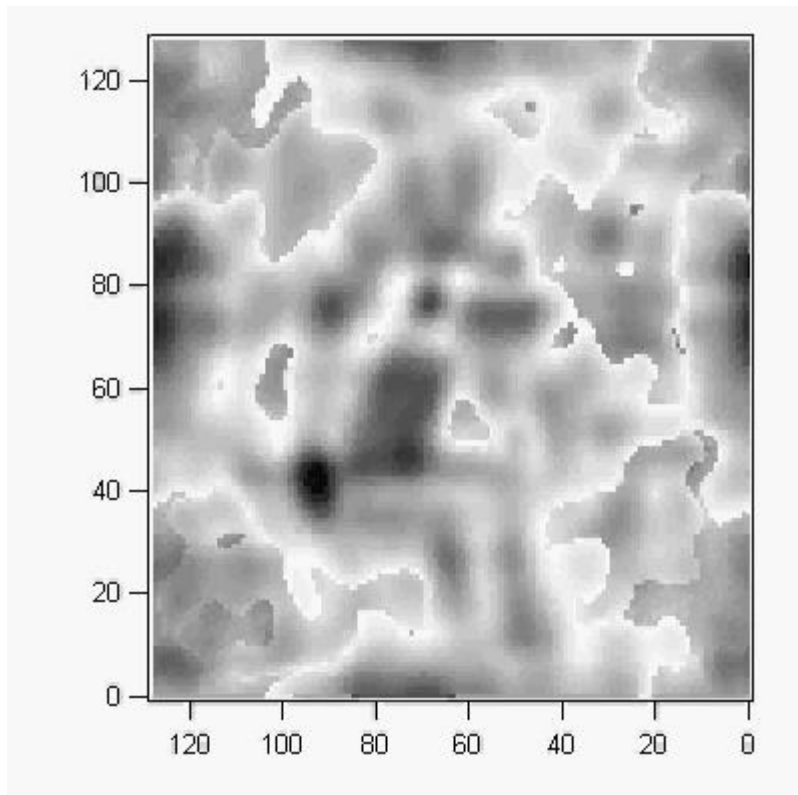


# Phase retrieval algorithm

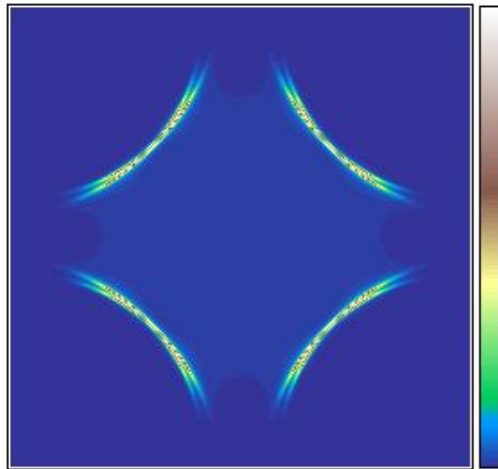


# Phase retrieval algorithm

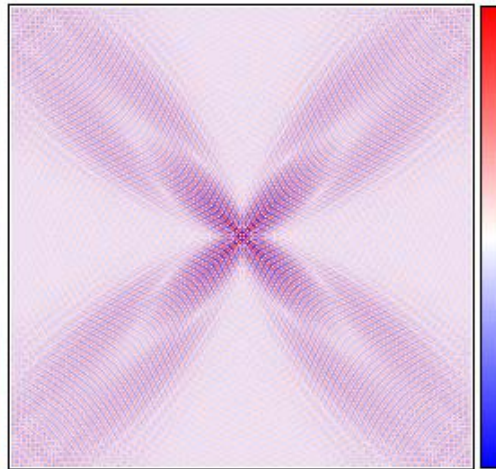
$$A(\mathbf{k}) = \text{PRA} |F A(\mathbf{k})|$$



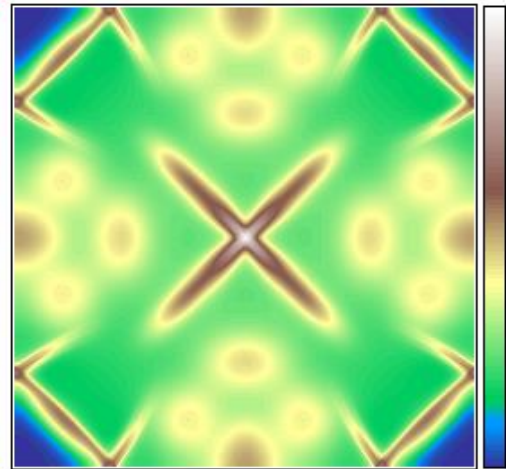
Representations of quasiparticles in different spaces of high- $T_c$  cuprates:



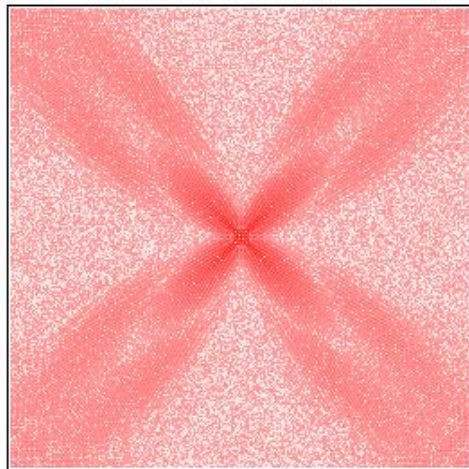
$A$  vs  $k$



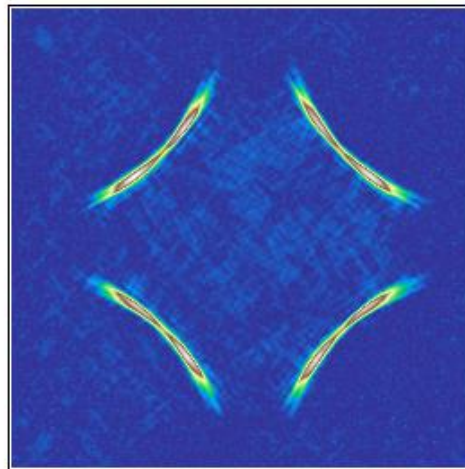
$FA$  vs  $\rho$



$ACA$  vs  $q$



$|FA| + 50\%$  noise vs  $\rho$

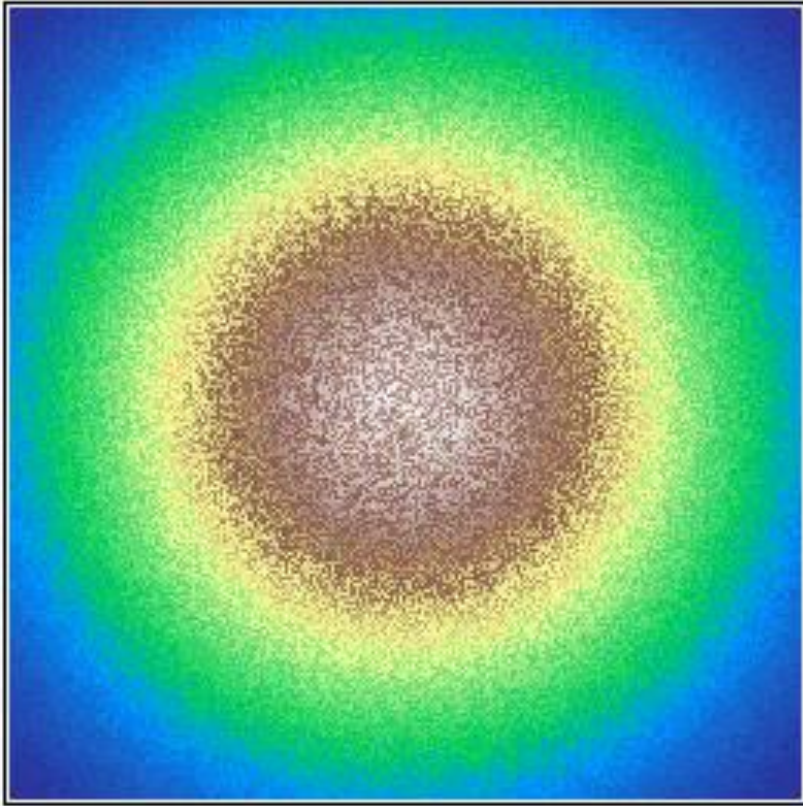


$A'$  vs  $k$



$ACA'$  vs  $q$

# Phase retrieval algorithm



$$\mathcal{R}_n = \mathbf{F}A_n,$$

$$\mathcal{R}'_n = R \exp[i\arg(\mathcal{R}_n)],$$

$$\mathcal{A}'_n = \mathbf{F}^{-1}\mathcal{R}'_n,$$

$$A_{n+1} = \begin{cases} \operatorname{Re}(\mathcal{A}'_n) & \text{if } \operatorname{Re}(\mathcal{A}'_n) \geq 0, \\ \operatorname{Re}(A_n - \beta\mathcal{A}'_n) & \text{if } \operatorname{Re}(\mathcal{A}'_n) < 0, \end{cases}$$