

Exercises to lecture 1 on 13.2.2017

Let $\Gamma \subset SL(2, \mathbb{R})$ be a discrete subgroup and $k \in \mathbb{Z}$. $SL(2, \mathbb{R})$ acts on $\mathcal{H}(\mathbb{H})$ via

$$(f|_k \gamma)(z) = (cz+d)^{-k} f\left(\frac{az+b}{cz+d}\right), \quad f \in \mathcal{H}(\mathbb{H}), \quad \gamma = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \in SL(2, \mathbb{R})$$

Denote by $\Omega_k(\Gamma)$ the \mathbb{C} -vector space

$$\Omega_k(\Gamma) = \left\{ f \in \mathcal{H}(\mathbb{H}) \mid \forall \gamma \in \Gamma \quad f|_k \gamma = f \right\}.$$

Let $f \in \Omega_k(\Gamma)$, $g \in \Omega_l(\Gamma)$. Then

Exercise 24

$$[f, g]_1 \stackrel{k, l \text{ def}}{=} -kfg' + lf'g \in \Omega_{k+l+2}(\Gamma).$$

Exercise 25 $[f, g]_2 \stackrel{k, l \text{ def}}{=}$

$$= \frac{k(k+1)}{2} fg'' - (k+1)(l+1) f'g' + \frac{l(l+1)}{2} f''g \in \Omega_{k+l+4}(\Gamma).$$